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GAUSS AND ITERATION METHODS FOR SOLVING A SYSTEM OF LINEAR ALGEBRAIC EQUATIONS

Annotation. This article describes the use of Gaussian and iteration methods in solving the system of linear algebraic equations, methods of forming students' knowledge of linear algebraic equations.

Keywords: simple and iterative methods, triangular matrix, forward and backward path, Hermite matrix. Diagonal elements, approximation to zero, number of iterations, speed of convergence, conditions of convergence, calculation algorithm in an iterative method, program text.

The occurrence of every observed event in life is subject to certain laws. The occurrence of such events is related to clearly taken into account factors, and their numerical relations have a specific character. One such relationship is a system of equations. Methods for solving systems of linear algebraic equations (CHATS) occupy an important place among numerical methods. The main reason for this is that many issues of the national economy are related to solving such systems. Therefore, in this topic, the methods of solving CHATS, the essence of exact and iterative methods, their calculation algorithms, and software are provided. Problems suitable for each calculation method were solved as examples, and the results were analyzed.

In practice, many problems lead to the solution of a system of linear equations. A number of problems in the design of engineering structures, processing of measurement results, solving the issue of planning the production process, and conducting technical, economic, and scientific experiments lead to the solution of the system of linear equations.

$$\begin{cases} a_{11}x_1+a_{12}x_2+\ldots+a_{1n}x_n=b_1 \\ a_{21}x_1+a_{22}x_2+\ldots+a_{2n}x_n=b_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{m1}x_1+a_{m2}x_2+\ldots+a_{mn}x_n=b_m \end{cases} \qquad (1)$$

A system of joint equations is said to be defined if it has a unique solution, and if it has two different solutions, it is said to be undefined.

Numerical methods of linear algebra include numerical methods such as solving a system of linear algebraic equations, finding the inverse of a matrix, and calculating determinants.

Let us be given this system of n linear algebraic equations of order n:

$$\begin{cases} a_{11}x_1+a_{12}x_2+\ldots+a_{1n}x_n=b_1 \\ a_{21}x_1+a_{22}x_2+\ldots+a_{2n}x_n=b_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{n1}x_1+a_{n2}x_2+\ldots+a_{nn}x_n=b_n \end{cases} \qquad (2)$$

$$A=\begin{pmatrix} a_{11}a_{12}\ldots a_{1n} \\ a_{21}a_{22}\ldots a_{2n} \\ \ldots\ldots\ldots\ldots\ldots \\ a_{n1}a_{n2}\ldots a_{nn} \end{pmatrix}, \qquad X=\begin{pmatrix} x_1 \\ x_2 \\ \ldots \\ x_n \end{pmatrix}, \qquad B=\begin{pmatrix} b_1 \\ b_2 \\ \ldots \\ b_n \end{pmatrix} (3)$$

Using the property of matrix and vector multiplication, we write the system (2) in matrix form, taking into account the designations (3):

A·X=B (4)

$$D=\det A \quad \begin{vmatrix} a_{11}\ a_{12}\ldots a_{1n} \\ a_{21}\ a_{22}\ldots a_{2n} \\ \ldots\ldots\ldots\ldots\ldots \\ a_{n1}\ a_{n2}\ldots a_{nn} \end{vmatrix} \neq 0. \qquad (5)$$

If D=0, the systems are called special systems and their solutions either do not exist or are infinitely many. The methods of solving the system of linear algebraic equations are divided into two groups: exact (exact) and iterative (approximate) methods. With proper methods, the solution of the system is achieved by performing a finite number of exact arithmetic operations. These methods are capable of solving a wide class of systems. But, at the same time, they are not without some shortcomings. For example, when they are used in a computer, all system coefficients and free terms must be stored in a memory device. In addition, despite the fact that the algorithms underlying the methods are exact, the solution is found to a certain extent approximate. Because rounding errors are always accumulated in successive calculation steps. Especially for high-order and ill-conditioned systems, this can lead to completely invalid solutions. Therefore, proper methods are used to solve well-conditioned, low-order, non-sparse matrix systems.

Iterative methods are successive approximation methods. These methods are more complicated than the correct methods. But, in most cases, it is better to use iterative methods. Because, when using these methods, there is no need to store all terms of the system matrix in the memory device of the computer. In addition, errors do not accumulate in iterative methods. At each iteration step, the calculation continues as if starting from scratch. However, iterative methods cannot be used all the time. For this, certain conditions must be met. Otherwise, the iteration process will be long-winded, and it will not be possible to obtain a sufficiently accurate solution. Detailed information about these conditions is given in the paragraph on iterative methods. Valid methods include Kramer, Gaussian, principal elements, square roots, and similar methods. Iterative methods include simple iteration, Seidel, relaxation, and similar methods.

The Gaussian method consists of a general scheme of the method of sequential elimination of system unknowns known to us from a simple mathematics course. Let us be given a system of n-order linear algebraic equations expressed in the form (2).

The Gaussian method consists of two steps: successive loss - straight walk and reverse walk.

At the correct walking stage of the method, the "rectangular" system of the form (2) is transformed into a "high triangle". In the reverse step, the generated "triangular" system is solved sequentially from the last equation upwards, and numerical solutions of the system are generated.

Straight walking stage. Let's assume that the leading element of the first equation in the system (2) is a_11≠0, otherwise, by changing the places of the equations in the system, we move the equation to the first place, the coefficient of which is different from zero in front of the unknown x_1. Divide all the coefficients of the first equation in the system by a_11,

$$x_1+a_{12}^{(1)}x_2+a_{13}^{(1)}x_3+\ldots+a_{1n}^{(1)}x_n=b_1^{(1)} \qquad (6)$$

$$\begin{cases} x_1+a_{12}^{(1)}x_2+a_{13}^{(1)}x_3+\ldots+a_{1n}^{(1)}x_n=b_1^{(1)} \\ 0+a_{22}^{(2)}x_2+\ldots+a_{2n}^{(2)}x_n=b_2^{(2)} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ 0+a_{n2}^{(2)}x_2+\ldots+a_{nn}^{(2)}x_n=b_n^{(2)} \end{cases} \qquad (7)$$

$$\begin{cases} a_{11}x_1+a_{12}x_2+\ldots+a_{1n}x_n=b_1 \\ a_{22}^{(2)}x_2+\ldots+a_{2n}^{(2)}x_n=b_2^{(2)} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{nn}^{(n)}x_n=b_n^{(n)} \end{cases} \qquad (8)$$

$$a_{ml}^{(k+1)}=a_{ml}^{(k)}-\frac{a_{mk}^{(k)}}{a_{kk}^{(k)}}a_{kl}^{(k)},\, b_m^{(k+1)}=b_m^{(k)}-\frac{a_{mk}^{(k)}}{a_{kk}^{(k)}}b_k^{(k)} \qquad (9)$$

$$k<m,l\leq n,1\leq k<n-1.$$

$$\begin{cases} x_n=b_n^{(n)}/a_{nn}^{(n)} \\ x_{n-1}=\left(b_{n-1}^{(n-1)}-a_{n-1,n}^{(n-1)}*x_n\right)/a_{n-1,n-1}^{(n-1)} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ x_1=b_1^{(1)}-a_{12}^{(1)}x_2-a_{13}^{(1)}x_3-\ldots-a_{1n}^{(1)}x_n \end{cases} \qquad (10)$$

Formulas for determining system solutions in the form (10) can be written in the following compact form:

$$x_n=\frac{b_n^{(n)}}{a_{nn}^{(n)}},\, x_k=\frac{1}{a_{kk}^{(k)}}\left(b_k^{(k)}-\sum_{i=k+1}^{n}a_{ki}^{(k)}*x_i\right)(11)$$

$$k=n-1,n-2,\ldots,1.$$

Thus, it is possible to solve the system of arbitrary n-order linear algebraic equations according to the indicated algorithm. Only the leading terms need to make different from zero or at least as large as modulo it. For this, the Gaussian method is used by selecting the leading term, that is, the equation with the largest coefficient in terms of modulus from the unknown loss column is selected as the working equation.

Iteration, that is, the method of successive approximation, is useful for solving high-order systems. In addition, unlike exact methods, this method has the property of non-accumulation of errors, which can be one of the decisive factors in solving high-order systems.

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