GEOMETRIK MASALALARNI YECHISHDA TENGSIZLIKLARNI QO'LLASH USULLARI

Jizzax politexnika instituti
Assistant o'qituvchisi
Tursunov Muzaffar Ilxomovich

Annotatsiya: Ma'lumki, elementar matematika kursida geometriya fani shartli ravishta ikki qismga bo'linadi: planimetriya va stereometriya. Geometriya bo'yicha ba'zi masalalarni, masalan, eng katta yoki eng kichik miqdorlarni topishga oid masalalarni yechish uchun ko'p jihatdan ularni yechish tajribasiga va yechish usullarini o'zlashtirganlik darajasiga bog'liq. Shuning uchun berilgan qarab qaysi tengsizlikni qo'llash va uni isbotlash muhimdir. Endi quyida ba'zi masalalarni qarab chiqaylik.

Kalit so'zlar: Geometriya, segment ,vatar, yoy, tengsizlik, tranzitivlik, mediana, o'rtacha qiymat, o'rta arifmetik, o'rta geometrik ,Koshi tengsizligi.

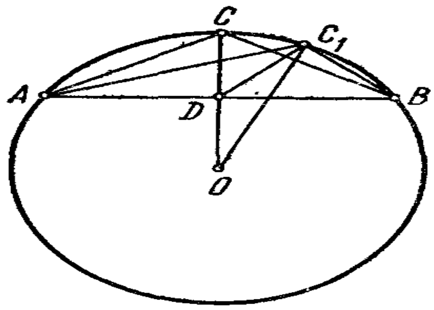
Jizzakh Polytechnic Institute
Assistant teacher
Tursunov Muzaffar Ilkhomovich

Annotation: As is known, in the elementary mathematics course, the subject of geometry is conventionally divided into two parts: planimetry and stereometry. Solving some problems in geometry, for example, problems related to finding the largest or smallest quantities, largely depends on the experience of solving them and the level of mastery of the solution methods. Therefore, it is important to apply which inequality to use and prove it in a given situation. Now let's look at some problems below.

Keywords: Geometry, segment, tangent, arc, inequality, transitivity, median, average value, arithmetic mean, geometric mean, Cauchy inequality.

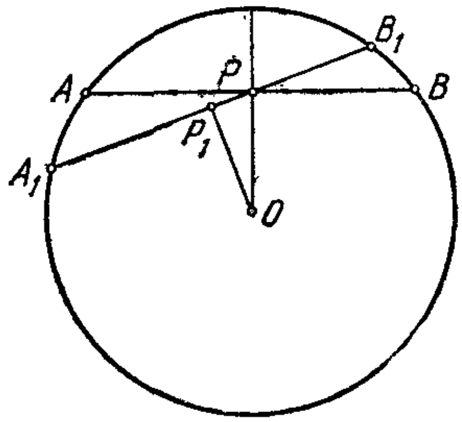
1-masala. Barcha uchburchaklarda asosi va uchidagi burchagi o'tmasligi ma'lum bo'lsa, uchburchak faqat teng yonli bo'lgandagina uchidan chiqqan mediana eng kichik bo'lishini ko'rsating.

Isbot. Izlanayotgan uchburchak asosi - AB da o'tmas burchak sig'adigan qilib segment yasaymiz (7-chizma) $ACB -$teng yonli uchburchak bo'lsin, $AC_1B -$ ixtiyoriy uchburchak, C va C_1 AB yoydagi nuqtalar. CD mediana C_1D medianadan kichik emasligini isbotlash kerak. Chizmadan ko'rinadiki $OD + DC = OC_1$ va $OC_1 < OD + DC_1$. Ikki munosabatni qo'shib $OD + DC + OC_1 < OC_1 + OD + DC_1$. Bundan $DC < DC_1$ kelib chiqadi. Isbotlandi.



(1-chizma)

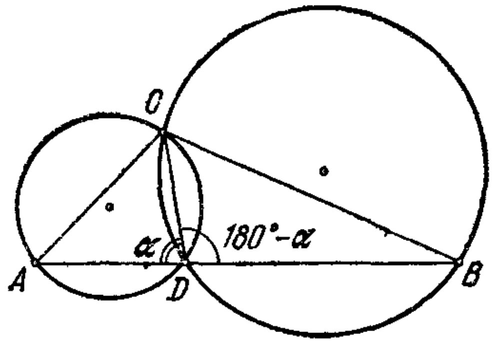
2-masala.Aylanada olingan nuqta orqali o'tgan eng kichik uzunlikka ega vatarni toping.
Yechish. P nuqta orqali 2 ta vatar : AB diametrga perpendikulyar va $A_1B_1 -$ ixtiyoriy P nuqtadan o'tuvchi vatar o'tkazamiz (2-chizma).



(2-chizma).

A_1B_1 ga OP_1 perpendikulyar o'tkazamiz. Hosil bo'lgan OP_1P to'g'ri burchakli uchburchakda OP gipotenuza OP_1 katetdan katta. Shunday qilib, $AB < A_1B_1$.
Demak, P nuqtadan o'tuvchi eng kichik vatar mavjud va u diametrga perpendikulyar .

3-masala ABC uchburchakning AB asosida yoki uning davomida D nuqta olingan va ACD, BCD uchburchaklarga aylana tashqi chizilgan. Agar ABC uchburchakning balandligi $-CD$ bo'lsa, aylana eng kichik radiusga ega bo'lishini isbotlang.
Isbot. 3-chizmani yasaymiz.



(3-chizma).

$ACD\ va\ BCD$ uchburchaklarga tashqi chizilgan radiusi mos ravishda R_1, R_2 bo'lsin va $< ADC = \alpha$ bo'lsin. $AC = 2R_1 \sin \propto$,

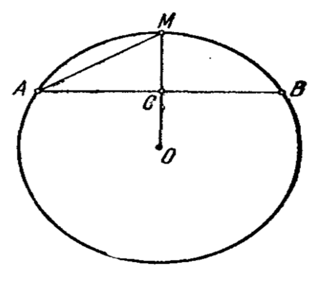
$$BC = 2R_2 \sin(180° - \alpha) = 2R_2 \sin \propto.$$

Bu yerdan $R_1 = \frac{AC}{2\sin\propto}$, $R_2 = \frac{BC}{2\sin\propto}$.

BC va AC doimiy kataliklar, $\sin \propto$ ning eng katta qiymati 1 ga teng. Bundan $\propto = 90°$.Demak, CD tomon ABC uchburchakning balandligi.

4-masala.Berilgan aylanaga ichki chizilgan muntazam $n-$burchak va $2n-$ burchakning tomonlari a_n va a_{2n} bo'lsa, $a_{2n} < \frac{2}{3}a_n$ tengsizlikni isbotlang.

Isbot. $AB = a_n$, $AM = a_{2n}$ va AB ning o'rtasi C nuqta bo'lsin. AMB yoy $\frac{360°}{n}$, MB yoy $\frac{180°}{n}$ dan tuzilgan.(4-chizma).



(4-chizma)

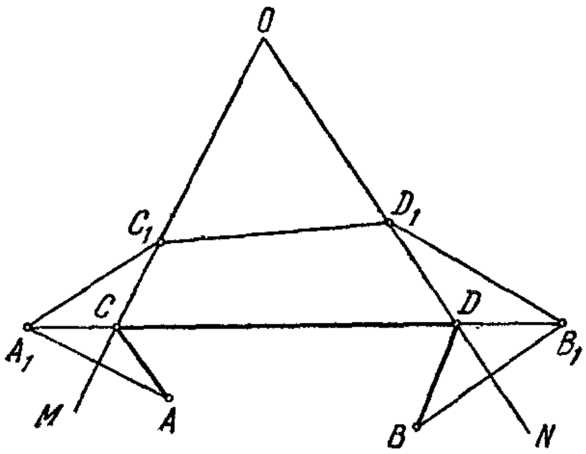
Biz $AM = a_{2n} = \frac{AM}{\cos(\angle MAB)} = \frac{AB}{2\cos(\angle MAB)} = \frac{a_n}{2\cos(\angle MAB)}$

ga egamiz. Demak, $a_{2n} = \frac{a_n}{2\cos(\angle MAB)}$

$n \geq 3$ da $a_{2n} \leq \frac{a_n}{2\cos 30^\circ} = \frac{a_n}{\sqrt{3}}$ va $\frac{1}{\sqrt{3}} < \frac{2}{3}$ ekanligidan $a_{2n} < \frac{2}{3}a_n$ kelib chiqadi.

5-masala. MON burchakning ichida A va B nuqtalar berilgan. OM va ON da mos ravishda yotuvchi C va D nuqtalarni topingki, $ACDB$ siniq chiziq eng kichik uzunlikka ega bo'lsin.

Yechish. 5-chizmani chizamiz.



(5-chizma).

OM va ON to'g'ri chiziqlarga nisbatan A va B nuqtalarni akslantiramiz. A nuqtaning aksi- A_1, B nuqtaning aksi- B_1 bo'lsin. A_1B_1 to'g'ri chiziq OM va ON lar bilan mos ravishda C va D nuqtalarda kesishadi. C va D biz izlayotgan nuqta ekanligi isbotlaymiz. Haqiqatan ham, $AC = A_1C$ va $BD = B_1D$ bo'lsa,

$$A_1B_1 = AC + CD + DB$$

va istalgan oxiri A_1 va B_1 nuqtalarda bo'lgan siniq chiziqdan kichik.Masalan, $A_1C_1D_1B_1$ siniq chiziqdan. Shunday qilib, $ACDB$ siniq chiziq $A_1C_1D_1B_1$ siniq chiziqdan kichik.

Foydalanilgan adabiyotlar

1.Sh. Ismoilov, A. Qo'chqorov, B. Abdurahmonov. Tengsizliklar-I Isbotlashning klassik usullari/Toshkent, 2008y.

2. Sh. Ismoilov, A. Qo'chqorov, B. Abdurahmonov. Tengsizliklar-II Isbotlashning klassik usullari/Toshkent, 2008y.

3.I.X. Sivyashenskiy.Neravenstva v zadachax.Moskva.1966y

4.Mathematical Olympiads,Problems and solutions from around the world,1998-1999.Edited by Andreescu T. and Feng Z. Washington 2000

5.Math Links, http://www.mathlinks.ro

6.Math Pro Press, http://www.mathpropress.com

7.Ayupov Sh., Rihsiyev B., Quchqorov O.”Matematika olimpiada masalalari”1,2 qismlar.T.:Fan,2004.