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DEKART KOORDINATALARI SISTEMASI VA SFERIK KOORDINATALI SISTEMASI ORASIDAGI BOG'LANISH

Annotatsiya: Ushbu maqolada sferik koordinatalar sistemasining ayrim qo'llanishlari bayon etilgan. Xususan, sfera tenglamasi, koordinata tekisliklari tenglamalari keltirilgan, hamda uch karrali integralni hisoblashda sferik koordinatalardan foydalanishga oid misol yechib ko'rsatilgan.

Kalit so'zlar: Dekart koordinatalar, sferik koordinatalar, sfera, uch karrali integral.

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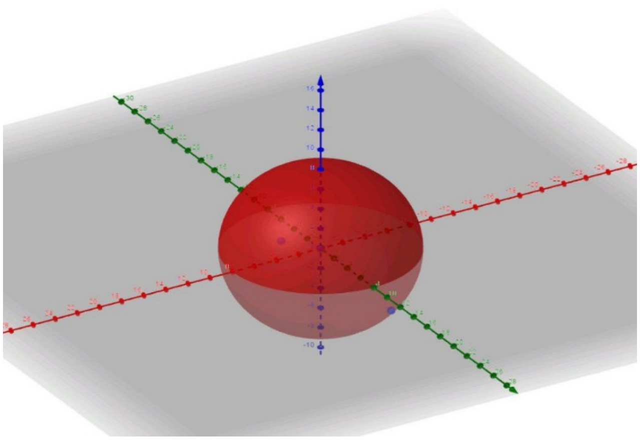
RELATIONSHIP BETWEEN THE CARTESIAN COORDINATE SYSTEM AND THE SPHERICAL COORDINATE SYSTEM

Abstract: This article describes some applications of the spherical coordinate system. In particular, the equation of the sphere, the equations of the coordinate planes are presented, and an example of the use of spherical coordinates in the calculation of the triple integral is shown.

Key words: Cartesian coordinates, spherical coordinates, sphere, triple integral.

Fazoda $Oxyz$ dekart koordinatalari sistemasi kiritilgan bo'lsin. Markazi koordinatalar boshida (ya'ni Onuqtada) bo'lgan R radiusli sferani qaraylik.



Ma’lumki, bu sferaning nuqtalari

$$x^2+y^2+z^2=R^2$$

tenglama bilan aniqlanadi. Sferadagi biror (ixiyoriy ravishda tanlangan) A nuqtaning Oxytekisligiga proyeksiyasi A' nuqta bo‘lsin. OA' Kesma Ox o‘qi bilan φ burchak hosil qilsin. OA va OA'kesmalar orasidagi burchak esa ψ bo‘lsin. U holda chizmadan Anuqtaning x_A, y_Ava z_A koordinatalari va R_A, φ_A, ψ_A kattaliklar orasida

$$x_A=R_A\cos\varphi_A\cos\psi_A,$$

$$y_A=R_A\sin\varphi_A\cos\psi_A,$$

$$z_A=R_A\sin\psi_A$$

bog‘lanishlar mavjudligini ko‘rish mumkin. $\sin$ va $\cos$ funksiyalar davriy bo‘lganligi uchun bu bog‘lanishlar o‘zaro bir qiymat bo‘la olmaydi. O‘zaro bir qiymatlilikni saqlash maqsadida

$$0\le\varphi<2\pi,$$

$$\frac{-\pi}{2}\le\psi\le\frac{\pi}{2}$$

cheklovlar kiritiladi. Anuqta ixtiyoriy ekanligidan qaralayotgan sferadagi har qanday (x,y,z) koordinatali nuqta

$$x=R\,cos\varphi\,sin\psi$$

$$y=R\,sin\varphi\,cos\psi$$

$$z=R\,sin\psi$$

$$0\le\varphi<2\pi,$$

$$\frac{-\pi}{2}\le\psi\le\frac{\pi}{2}$$

munosabatlarni qanoatlantiradi.

Hosil qilingan $\{O,R,\varphi,\psi\}$ sistema fazodagi sferik koordinatalari sistemasi deyiladi. Bunda yuqoridagi munosabatlar sferik koordinatalar R,φ,ψdan Dekart koordinatalar x,y,z ga o‘tish formulalari deb yuritiladi.

$\{O,R,\varphi,\psi\}$ koordinatalar sfera orqali kiritilgani uchun sferik koordinatalar sistemasi deb yuritiladi.

Boshqacha izoh: Fazoda tayinlangan O nuqta va $R>0$ kattalik o'zgarmasa φ va ψ kattaliklar qabul qilishi mumkin bo'lgan barcha qiymatlarni qabul qilganda hosil bo'lgan nuqtalar to'plami fazodagi sferani beradi.

Shuni alohida takidlash joizki, $R=0$ bo'lganda φ va ψ kattaliklarning har qanday qiymatida ham yagona O nuqta hosil bo'laveradi. Shuning uchun odatda $R>0$ qiymatlar qaraladi.

Endi $Oxyz$ fazodagi koordinatalar bilan berilgan nuqtaning R,φ,ψsferik koordinatalarini topamiz. Chizmada ko'rinadiki,

$$R=\sqrt{x^2+y^2+z^2},$$

$$\varphi=\arccos\frac{x}{\sqrt{x^2+y^2}}, \text{ yoki } \varphi=\arcsin\frac{y}{\sqrt{x^2+y^2}}$$

$$\psi=\arcsin\frac{z}{\sqrt{x^2+y^2+z^2}}, \text{yoki } \psi=\arccos\frac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}}$$

(2) munosabatlar dekart koordinatali sistemasidan sferik koordinatalar sistemasiga o'tish formulalari deb yuritiladi.

Ayrim fazoviy figuralarning sferik koordinatalar sistemasidagi tenglamalarni keltiramiz.

1. Markazi koordinatalar boshida bo'lgan, radiusi R_0ga teng sfera tenglamasi:

$$R=R_0$$

2. $x=0$tekislik (ya'ni, Oxzkoordinatalar tekisligi) tenglamasi:

$$\varphi=\frac{\pi}{2},\psi=\frac{3\pi}{2}$$

3. $y=0$tekislik tenglamasi (ya'ni Oxzkoordinata tekisligi) tenglamasi:
4. $z=0$ tekislik (ya'ni Oxykoordinata tekisligi) tenglamasi:

$$\psi=\frac{\pi}{2}$$

Endi sferik koordinatalar sistemasining uch karrali integrallarni hisoblashdagi tatbig'ini ko'rsatamiz. Buning uchun

$$\iiint\limits_{\Omega} f(x,y,z)dxdydz=\iiint f(R\cos\varphi\cos\psi, R\sin\varphi\cos\psi, R\sin\psi)\left|J\right|dRd\varphi d\psi$$

formuladan foydalaniladi. Bu yerda J Yakobian deb ataluvchiushbu determinantdan iborat:

$$J=\begin{vmatrix}\frac{\partial x}{\partial R} & \frac{\partial x}{\partial \varphi} & \frac{\partial x}{\partial \psi}\\ \frac{\partial y}{\partial R} & \frac{\partial y}{\partial \varphi} & \frac{\partial y}{\partial \psi}\\ \frac{\partial z}{\partial R} & \frac{\partial z}{\partial \varphi} & \frac{\partial z}{\partial \psi}\end{vmatrix}=\begin{vmatrix}\cos\varphi\cos\psi & -R\sin\varphi\cos\psi & -R\cos\varphi\sin\psi\\ \sin\varphi\cos\psi & R\cos\varphi\cos\psi & -R\sin\varphi\sin\psi\\ \sin\psi & 0 & R\cos\psi\end{vmatrix}=$$

$$=R^2\cos^2\varphi\cos^3\psi+R^2\sin^2\varphi\sin^2\psi\cos\psi+R^2\cos^2\varphi\sin^2\psi\cos\psi+$$
$$+R^2\sin^2\varphi\cos^3\psi=R^2\cos^3\psi+R^2\sin^2\psi\cos\psi=R^2\cos\psi$$

Demak,

$$\iiint\limits_{\Omega} f(x,y,z)dxdydz=\iiint f(R\cos\varphi\cos\psi,R\sin\varphi\cos\psi,R\sin\psi)R^2\cos\psi dRd\varphi d\psi$$

Misol.

$\iiint\limits_{\Omega}(x^2+y^2+z^2)dxdydz$ uch karrali integralni $\Omega: x^2+y^2+z^2\le 1$ shar bo‘yicha hisoblang.

Yechish. Avvalo $\begin{array}{l}x=R\cos\varphi\cos\psi\\ y=R\sin\varphi\cos\psi\\ z=R\sin\psi\end{array}$ tengliklar qo‘llab,

$$(x^2+y^2+z^2)=R^2\cos^2\varphi\cos^2\psi+R^2\sin^2\varphi\cos^2\psi+R^2\sin^2\psi=$$
$$=R^2\cos^2\psi+R^2\sin^2\psi=R^2$$

Ekanligini topamiz. Keyin Ω sharning sferik koordinatalar sistemasidagi ifodasini yozib olamiz, bunda shar radiusi 1 ekanligini e’tiborga olamiz:

$$0\le R\le 1,$$
$$0\le\varphi<2\pi,$$
$$\frac{-\pi}{2}\le\psi\le\frac{\pi}{2}$$

Shuning uchun

$$\iiint_{\Omega}(x^2+y^2+z^2)dxdydz=\iiint_{\Omega}R^2R^2\cos\psi dRd\varphi d\psi=\int_0^{2\pi}d\varphi\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos\psi d\psi\int_0^1R^4dR=$$

$$=2\pi*2*\frac{R^4}{5}\Big|_0^1=4\pi*\frac{1}{5}=\frac{4\pi}{5}.$$

hosil bo‘ldi.

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