SOME METHODS OF INTEGRATION OF IRRATIONAL FUNCTIONS

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Abstract. The article discusses the issues of teaching and learning mathematics. The integration of functions, which is one of the main concepts of mathematics, is taught to students in simple ways. For this, it is taught from the integration of the simplest functions to the integration of irrational functions.

Key words. Irrational number, irrational function, integral, binomial integral, Euler substitutions, integration of irrational functions, methods of integration.

1. ***Integration of irrational functions.*** If the function y=f(x) consists of an algebraic expression with fractional levels of the argument x, we call it an irrational function.

❖ D Originally called the binomial integral an

$$I(r,s,p)=\int x^{r}(a+bx^{s})^{p}\,dx$$

we look at the integrals that appear. Here r, s, p represent rational numbers and a, b represent real numbers. If all three numbers r, s, p are integers, then a rational function is formed under the integral, and in this case, the binomial integral is expressed in elementary functions. If at least one of the numbers r, s, p is not an integer, then an irrational function is formed under the binomial integral. It was proved by the great Russian mathematician P. L. Chebyshev (1821-1894) that the binomial integral can be expressed in elementary functions only in the following three cases:

1) p is an integer. In this case, we make a substitution (m is the common denominator of the numbers r and s under the integral). If we take r=k/m, s=q/m, then and is a binomial integral

$$I(r,s,p)=m\int t^{k+m-1}(a+bt^{q})^{p}\,dt$$

takes the form and comes to the integral obtained from the rational function. $I=\int\frac{dx}{x(1+\sqrt[3]{x})^2}$

we calculate the integral. This is a binomial integral with parameters r=–1, s=1/3 and p=–2, and calculating it using substitution based on the above, we get this result: $I=\int\frac{3t^2dt}{t^3(1+t)^2}=3\int\frac{dt}{t(1+t)^2}=3[\int\frac{dt}{t}-\int\frac{dt}{t+1}-\int\frac{dt}{(t+1)^2}]=$

$$=3\left[\ln|t|-\ln|t+1|+\frac{1}{t+1}\right]+C=3\left[\ln\left|\frac{\sqrt[3]{x}}{1+\sqrt[3]{x}}\right|+\frac{1}{1+\sqrt[3]{x}}\right]+C.$$

2) *n*=(*r*+1)/*s* – whole number. In this case, if p=k/m, then the substitution a+bxs=tm is used. In this

$$(a+bx^s)^p=t^k,\quad x^r=\left(\frac{t^m-a}{b}\right)^{\frac{r}{s}},\quad dx=\frac{m}{bs}\left(\frac{t^m-a}{b}\right)^{\frac{1}{s}-1}t^{m-1}dt$$

is, and the binomial integral comes to the following integral with rational fractions:

$$I(r,s,p)=\frac{m}{b^n s}\int(t^m-a)^{n-1}t^{k+m-1}dt.$$

3) *n*=*p*+(*r*+1)/*s* – whole number. In this case, if p=k/m, then The substitution ax–s+b=t is used. In this

$$x=\left(\frac{a}{t^m-b}\right)^{\frac{1}{s}},\quad (a+bx^s)^p=x^{ps}(ax^{-s}+b)^p=\left(\frac{a}{t^m-b}\right)^p t^k,$$

$$x^r=\left(\frac{a}{t^m-b}\right)^{\frac{r}{s}},\quad dx=-\frac{ma}{s}\left(\frac{a}{t^m-b}\right)^{\frac{1}{s}-1}\frac{t^{m-1}}{(t^m-b)^2}dt$$

$$I(r,s,p)=-\frac{ma^n}{s}\int\frac{t^{k+m-1}}{(t^m-b)^{n-1}}dt.$$

❖ $I=\int R(x,x^{\frac{m}{n}},\ldots,x^{\frac{r}{s}})dx$ we look at integrals of the form In this case, only rational operations are expressed with respect to the variables x, xm/n,..., xr/s included in R, and m, n, ..., r, s are natural numbers. To calculate this integral, we find the common denominator k of the fractional degrees participating in it and perform substitution. In this case, x, xm/n,..., xr/s fractional exponent degrees are represented by whole degrees of the new variable t, and as a result we create a rational fractional integral. Calculating this integral and taking the result as t=x1/n, we find the given indefinite integral. $I = \int R\left[x, (\frac{ax+b}{cx+d})^{\frac{m}{n}}, \ldots, (\frac{ax+b}{cx+d})^{\frac{r}{s}}\right]dx$ Let's look at the integral in the form Here the conditions set in the previous integral for R, m, n, s, r are preserved. For real numbers a, b, c and d in fractions, we set the condition a/b≠c/d, because if this condition is not fulfilled

$$\frac{ax+b}{cx+d} = \frac{b}{d} \cdot \frac{\frac{a}{b}x+1}{\frac{c}{d}x+1} = \frac{b}{d}$$

and the irrationality in the integral disappears.

If the common denominator of fractions m/n, … , r/s is k, then to calculate this integral

$$\frac{ax+b}{cx+d} = t^{\kappa}, \quad t = \sqrt[k]{\frac{ax+b}{cx+d}}$$

we will do the exchange. In this case

$$x = \frac{b - dt^m}{ct^m - a}, \quad dx = \frac{mt^{m-1}(ad-bc)}{(ct^m - a)^2}dt,$$

that is, x and dx are rationally represented by a new variable t. Therefore, as a result of the above substitution, we get the integral of the rational function for the given integral. By calculating this integral and replacing t with its above expression in the resulting result, we find the answer of the given integral I.

For integrals of the form Such integrals with an irrational expression are reduced to rational fractional integrals and calculated using the substitutions proposed by the great Swiss mathematician L. Euler (1707-1783). Three cases are considered here.

Case I. In this case, the considered IE is taken as a>0 in the integral. In this case, the integral from the variable x to the new variable t is called Euler's substitution I $t = x\sqrt{a} - \sqrt{ax^2 + bx + c} \Rightarrow \sqrt{ax^2 + bx + c} = x\sqrt{a} - t$

is passed through the visible switch. In this case, IE is expressed as a rational fraction through the new variable t in the integral x, and dx. So, the considered IE integral was reduced to a rational fractional integral, and the intended goal was achieved.

II hol. Now

$$I = \int \frac{dx}{x\sqrt{x^2 + 4x - [illegible]}}$$

Let c>0. In this case, we use this Euler substitution II to calculate the integralIE:

$$\sqrt{ax^2 + bx + c} = xt + \sqrt{c}\ .$$

As a result of this substitution, we arrive at a rational fractional integral. As an example, we calculate this integral:.

$$I = \int \frac{(1 - \sqrt{1 + x + x^2})^2}{x^2\sqrt{1 + x + x^2}} dx$$

According to Euler's substitution II, we get:

$$\sqrt{1 + x + x^2} = xt + 1 \Rightarrow 1 + x + x^2 = x^2t^2 + 2xt + 1 \Rightarrow x = \frac{2t - 1}{1 - t^2} \Rightarrow$$

$$dx = \frac{2t^2 - 2t + 2}{(1 - t^2)^2} dt\,, \quad \sqrt{1 + x + x^2} = xt + 1 = \frac{t^2 - t + 1}{1 - t^2} \Rightarrow \ \Rightarrow 1 - \sqrt{1 + x + x^2} = \frac{-2t^2 + t}{1 - t^2}\ .$$

We put these generated expressions into the given integral:

$$\int \frac{1 - \sqrt{1 + x + x^2}}{x^2\sqrt{1 + x + x^2}} dx = \int \frac{(-2t^2 + t)^2 (1 - t^2)^2 (1 - t^2)(2t^2 - 2t + 2)}{(1 - t^2)^2 (2t - 1)^2 (t^2 - 1)^2 (t^2 - t + 1)} dt =$$

$$= -\frac{2(\sqrt{1 + x + x^2} - 1)}{x} + \ln\left|\frac{x + \sqrt{1 + x + x^2} - 1}{x - \sqrt{1 + x + x^2} + 1}\right| + C\ .$$

III hol. The quadratic triangle under the considered IE integral has real roots ✓ and ✓, that is, let the discriminant be D=b2–4ac>0. In this case

$$\sqrt{ax^2+bx+c}=(x-\alpha)t$$

using Euler's III substitution in the form, we convert the expression under the integral into the form of a rational fraction.

Here the quadratic triangle 2+x–x2 has real roots ✓ =–1 and ✓ =2 and can be written as 2+x–x2 = (x+1) (2–x). Therefore, we use Euler's substitution III and from it we get the following equations:

$$\sqrt{2+x-x^2}=t(x+1)\Rightarrow\sqrt{(x+1)(2-x)}=t(x+1)\Rightarrow\sqrt{2-x}=t\sqrt{x+1}\ \Rightarrow$$

$$\Rightarrow 2-x=t^2(x+1)\Rightarrow x=\frac{2-t^2}{t^2+1}\Rightarrow dx=(\frac{2-t^2}{t^2+1})'dt=-\frac{6tdt}{(t^2+1)^2}.$$

$\sqrt{2+x-x^2}=t(\frac{2-t^2}{t^2+1}+1)=\frac{3t}{t^2+1}$ using the fact that , we calculate the above integral as follows: $\int\frac{dx}{x\sqrt{2+x-x^2}}=-6\int\frac{tdt}{\frac{2-t^2}{t^2+1}\cdot\frac{3t}{t^2+1}(t^2+1)^2}=-2\int\frac{dt}{2-t^2}=$

$$=-2\int\frac{dt}{2-t^2}=-\frac{1}{\sqrt{2}}\ln\left|\frac{\sqrt{2}+t}{\sqrt{2}-t}\right|+C=-\frac{1}{\sqrt{2}}\ln\left|\frac{(t+\sqrt{2})^2}{2-t^2}\right|+C=$$

$$=-\frac{1}{\sqrt{2}}\ln\left|\frac{(t+\sqrt{2})^2}{2-t^2}\right|+C=-\frac{1}{\sqrt{2}}\ln\left|\frac{t^2+2+2t\sqrt{2}}{2-t^2}\right|+C.$$

We express the fraction under the logarithm by x and, simplifying,

$$\int\frac{dx}{x\sqrt{2+x-x^2}}=-\frac{1}{\sqrt{2}}\ln\left|\frac{2\sqrt{2}\sqrt{2+x-x^2}+x+4}{3x}\right|+C$$

we get the result.

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