MAVZU: IKKI KARRALI INTEGRALLARNI CHEGARALARINI TANLASHDA GEOGEBRA DASTURIDAN FOYDALANISH.

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TOPIC: USING THE GEOGEBRA PROGRAM TO SELECT THE BOUNDARIES OF DOUBLE INTEGRALS.

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Annotatsiya: Ushbu maqolada ikki karrali integralni hisoblashda asosiy bajariladigan ishlardan eng avvalo integral chegarasini qo'yish va integrallash tartibini o'zgartirish masalalari ko'rib chiqiladi. Bundan tashqari ikki karrali integralni hisoblashda integral chegarasini qo'yishda soha chizish uchun Geogebra dasturi va uning foydali chihatlari ko'rib chiqiladi.

Kalit so'zlar: Ikki karrali integrallar, soha, integral chegarasi, integral chegarasini tanlash.

Abstract: In this article, the issues of setting the limit of the integral and changing the order of integration are considered among the main tasks in the calculation of the double integral. In addition, the Geogebra program and its useful aspects are considered for drawing the area when calculating the integral limit when calculating the double integral.

Key words: Double integrals, field, integral limit, choice of integral limit.

Asosiy qism: Oliy ta'lim muassasalari talabalariga matematik analizning ba'zi bo'lim va mavzularini o'qitishda bir muncha qiyinlilikka uchrash holatlari mavjudligini ko'pchilik ta'kidlashi mumkin. Buning sababi sifatida hozirgi kunda talabalarning soha tushunchasi va ba'zi sohalarni tasavvur qilish qobiliyati kamligi sababli ikki karrali integralni hisoblashda integrallash chegarasini qo'yishda juda ko'p kamchliklarga yo'l qo'yishadi. Shuni oldini olish maqsadida quyida biz integrallash chegarasini o'rnatishda Geogebra dasturidan foydalanish va uning samaralari to'g'risida to'xtalib tahlil qilib o'tamiz. Quyida bir nechta misollarni tahlil qilamiz.

Misol 1: Ushbu $y^2+8x=16,\ y^2-24x=48.$ chiziqlar bilan chegaralangan D soha uchun $\iint\limits_D f(x,y)dxdy$ ikki karrali integral takroriy integralga keltirilsin [1].

Yechish: Birinchi navbatda

$y^2+8x=16,\ y^2-24x=48.$

chiziqlarning kesishish nuqtalarini topamiz. Buning uchun

$$\begin{cases} y^2+8x=16 \\ y^2-24x=48 \end{cases}$$

tenglamalar sistemasini yechib olamiz [3,4]. Buning uchun maqktab matematika kursidan ma'lum bo'lgan tenglamalar sistemasini yechishning arifmetik qo'shish usulidan foydalanib tenglamalar sistemasining birinchi tenglamasidan ikkinchi tenglamasini ayiramiz va

$$32x=-32$$

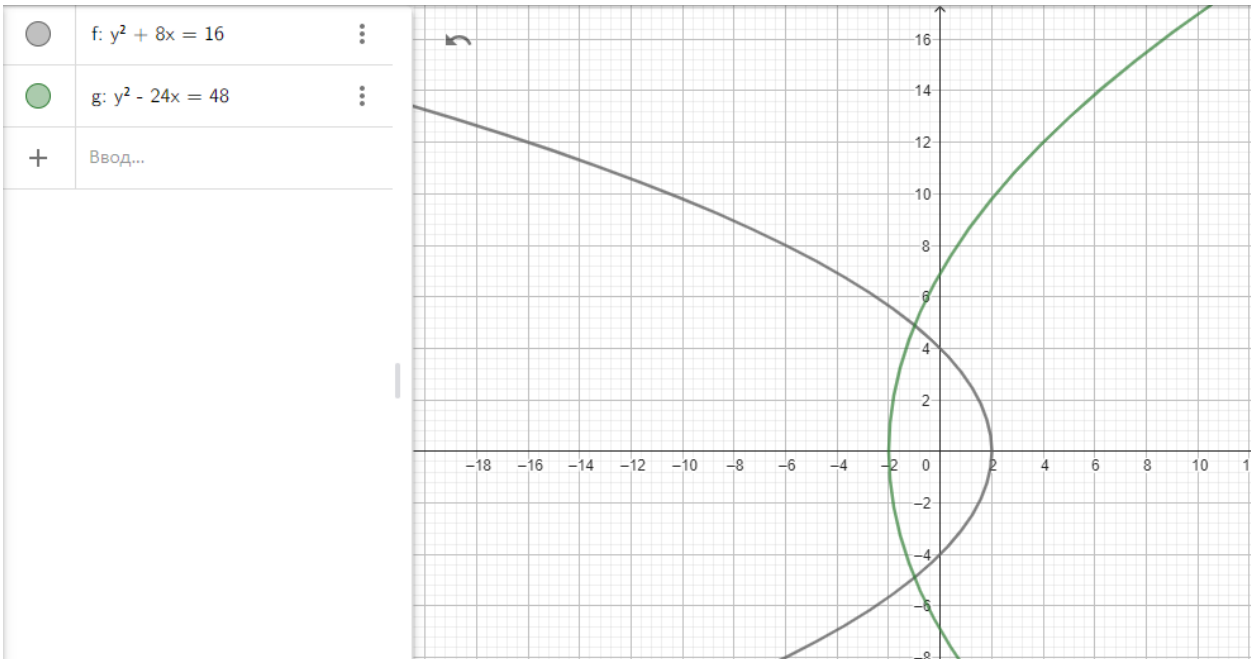
tenglik hosil bo'ladi. Bundan esa

$$x=-1,\ y=\pm 2\sqrt{6}$$

ekanligi kelib chiqadi. Ushbu funksiyalardan argumenti y o'zgaruvchiga bog'liq bo'lgan ikkita funksiyani keltirib olamiz

$$x=2-\frac{y^2}{8}, x=\frac{y^2}{24}-2.$$

Keyin takroriy integralning integrallanish sohasini aniqlash uchun keyingi navbatda sohani chizmada tasvirlaymiz.



Ushbu chizmadan sohani quyidagicha yozishimiz mumkin.

$$D=\left\{-2\sqrt{6}\le y\le 2\sqrt{6}, \frac{y^2}{24}-2\le x\le 2-\frac{y^2}{8}\right\}, [2]$$

bundan esa

$$\iint_D f(x,y)dxdy=\int_{-2\sqrt{6}}^{2\sqrt{6}}dy\int_{\frac{y^2}{24}-2}^{2-\frac{y^2}{8}}f(x,y)dx$$

Misol 2: $\int_0^1 dy\int_0^{y^3} fdx+\int_1^2 dy\int_0^{2-y} fdx.$ integrallash tartibini o'zgartiring [1]**.**

Yechish: Ushbu takroriy integralni integrallash tartibini o'zgartirish uchun eng avvalo berilgan integralning chegarasini grafigini chizib olamiz. Bundan tashqari

$$y^3=x, 2-y=x$$

funksiyadan y ni topamiz.

$$y=\sqrt[3]{x}, y=2-x$$

funksiyalarning o’zaro keisshish niqtalarini topamiz [4,5]. Buning uchun ikkita funksiyani tenglashtirib tenglama yechamiz.

$$\sqrt[3]{x}=2-x$$

bu yerda

$$\sqrt[3]{x}=t$$

belgilash kiritib olsak tenglamamiz,

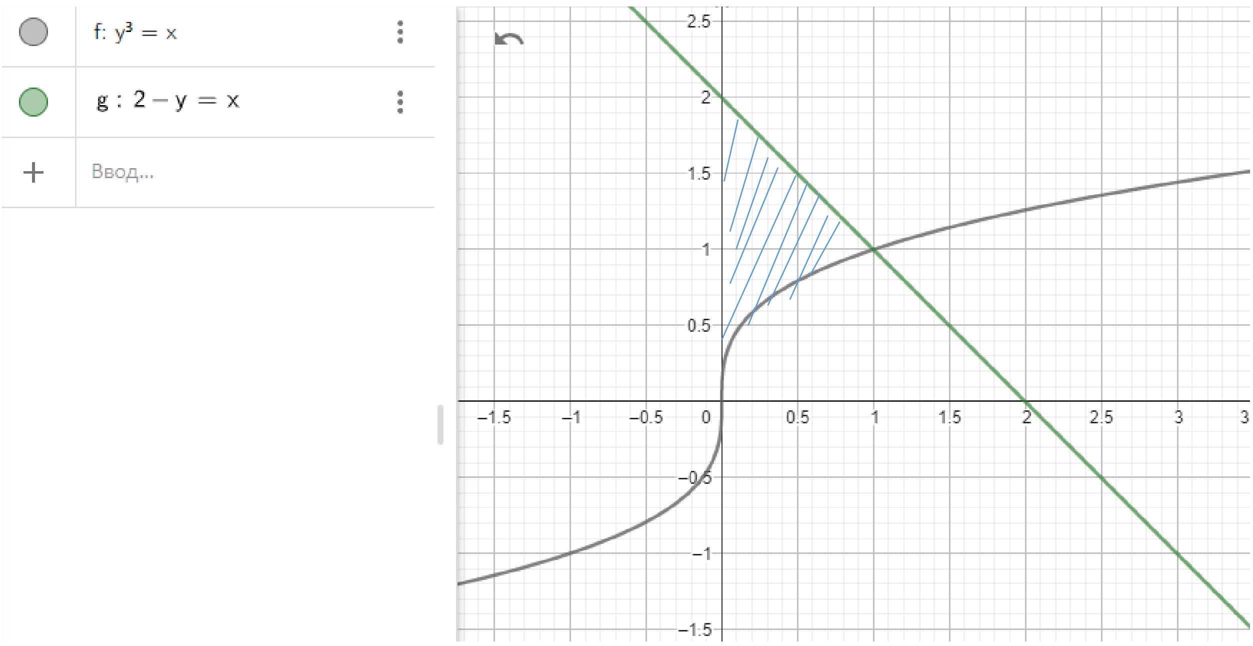
$$t^3+t-2=0$$

Ko’rinishga keladi. Endi ushbu tenglamani

$$t^3-1+t-1=0$$

$$(t-1)(t^2+t+2)=0$$

$$t=1 \rightarrow x=1$$



Ushbu grafikdagi shtrixlangan qism bo’yicha olingan integral berilgan va bu integralni tartibini o’zgartirish zarur. Buning uchun berilgan sohani quyadagicha tengsizliklar orqali yozib olamiz.

$D=\{0\le y\le 1, 0\le x\le y^3\}\cup\{1\le y\le 2, 0\le x\le 2-y\}$ va

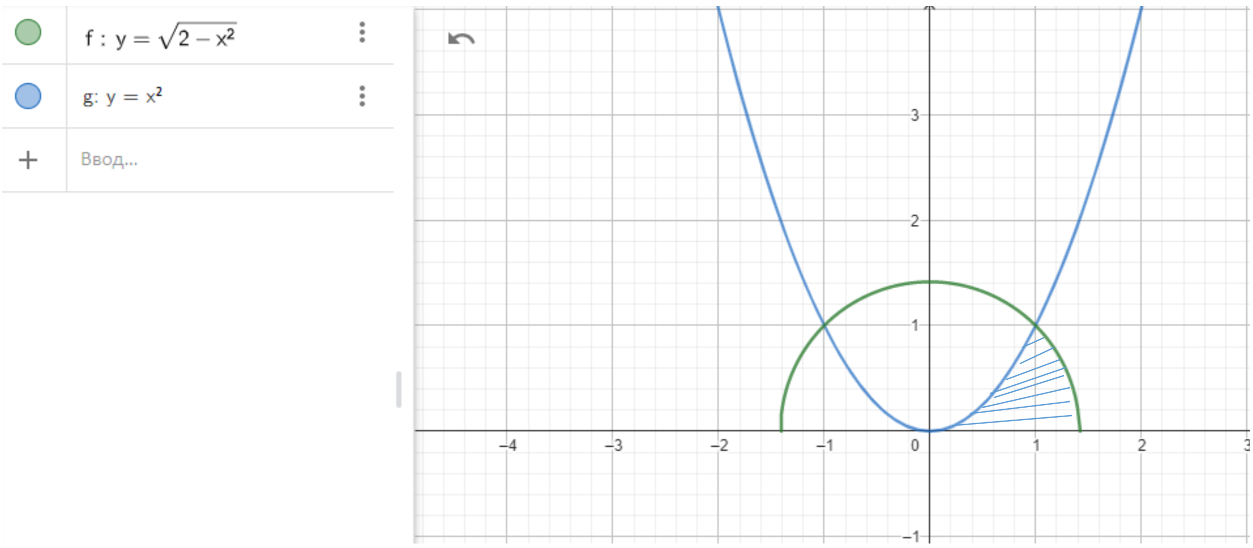
$$D=\{0\le x\le 1, \sqrt[3]{x}\le y\le 2-x\}$$

kabi yozib olamiz va yuqoridagi takroriy integralni quyidagicha yozishimiz mumkin.

$$\int_0^1 dy\int_0^{y^3} fdx+\int_1^2 dy\int_0^{2-y} fdx. ¿\int_0^1 dx\int_{\sqrt[3]{x}}^{2-x} f(x,y)dy$$

Misol 3: $\int_0^1 dx\int_0^{x^2} fdy+\int_1^{\sqrt{2}} dx\int_0^{\sqrt{2-x^2}} fdy$ integrallash tartibini o’zgartiring [1].

Yechish: Ushbu berilgan takroriy limitning ham integrallash tartibini o’zgartirish uchun huddi yuqoridagi misoldagi kabi berilgan integralning chegaralaridan iborat sohani chizib olamiz.



Ushbu shtrixlangan soha ikki karrali integralning integrallash sohasidir. Endi integrallsh tartibini o’zgartirish uchun, ikkita funksiyaning kesishish nuqtalarini va y ga bog’liq funksiyalarni topib olamiz. Buning uchun,

$$\sqrt{2-x^2}=x^2$$

tenglamani yechamiz.

$$2-x^2=x^4,\ x^4+x^2-2=0 \rightarrow x^2=t\ [2,3]$$

belgilash kiritib olamiz.

$$t^2+t-2=0$$

tenglama hosil bo'ladi. Bu tenglamaning yechimlari,

$$t_1=-2, t_2=1 \rightarrow x_{1,2}=\pm 1$$

bo'lib, endi yuqoridagi ikkita funksiyadan

$$y=\sqrt{2-x^2}, y=x^2$$

lardan, x ni topib olamiz,.

$$x=\sqrt{2-y^2}, x=\sqrt{y}$$

va sohani quyidagicha yozib olamiz,

$$D=\{0\leq x\leq 1, 0\leq y\leq x^2\}\cup\{1\leq x\leq\sqrt{2}, 0\leq y\leq\sqrt{2-x^2}\}, D=\{0\leq y\leq 1, \sqrt{y}\leq x\leq\sqrt{2-y^2}\}$$

shulardan kelib chiqib, ikki karrali integrallarning integrallash tartibini quyidagicha yozib olamiz.

$$\int_0^1 dx\int_0^{x^2} fdy+\int_1^{\sqrt{2}} dx\int_0^{\sqrt{2-x^2}} fdy \text{¿} \int_0^1 dy\int_{\sqrt{y}}^{\sqrt{2-y^2}} f(x,y)dx.$$

Xulosa: Yuqoridagi misollardan ko'rinadiki, oliy ta'lim muassasalari talabalariga ikki karrali integrallarni o'rgatishda Geogebra va shunga o'xshgan dasturlardan foydalanib sohalarini shizish orqali o'rgatish nafaqat misol yechimini chiqarishda balki talabalarga tasavvur qobiliyatini rivojlantirishda ham juda o'rni katta bo'lib hizmat qiladi. Chunki shunday funksiyalar borki talaba bir ko'rishining o'zidayoq bu funksiya grafigi to'g'risida yetarli xulosaga kela olmasligi mumkin. Shu borada Geogebra va shunga o'xshagan dasturlar, bir muncha qiyin funksiyalar va ularning grafiklari hususida xulosa qilishga va karrali integrallarni, nafaqat karrali integrallar balki bir nechta funksiya bilan chegaralangan soha yuzasini topishda integraldan foydalanishda ham qo'l keladi.

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