# DIFFERENTSIAL-FUNKTSIONAL TENGLAMALAR 

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Annotation: In the article, the concepts of the existence and uniqueness of the solution of the equation with constant coefficients, as well as the dependence on the initial value or function, are fully studied. Bruvy series is considered as an application in determining the solution of linear, inhomogeneous or homogeneous differential-functional equations with constant and variable coefficients.

Key words: differential functional equation, field of real numbers, Stilthies integral, matrix function, finite difference differential.

Definition. 1.1. According to the equality given in the form below, is finite differential differential and differential-functional equation are called: $\dot{X}(t)=F\left(t, X(t), X\left(t-r_{1}\right), \ldots, X\left(t-r_{n}\right)\right.$,

$$
\begin{equation*}
\left.\dot{X}\left(t-\tau_{1}\right), \dot{X}\left(t-\tau_{2}\right), \ldots, \dot{X}\left(t-\tau_{m}\right)\right) \tag{1.1}
\end{equation*}
$$

and

$$
\begin{equation*}
\dot{X}(t)=F\left(t, \int_{0}^{r} X(t+\theta) d \xi(\theta), \int_{0}^{\tau} \dot{X}(t+\theta) d \eta(\theta,)\right) \tag{1.2}
\end{equation*}
$$

here "•" sign $\dot{X}(t)=\frac{d X(t)}{d t}$ means, ${ }^{r}, \tau_{s}, r$ and $\tau$ consists of some elements of the field of real numbers, and all $i$ and $S$ for s

$$
\begin{gathered}
r_{i} \neq 0, \tau_{s} \neq 0, r \neq 0, \tau \neq 0, r_{i} \neq r_{j}, \tau_{s} \neq \tau_{k} \\
(i=1,2, \ldots, n ; j=1,2, \ldots, n ; s=1,2, \ldots, m ; j=1,2, \ldots, m)
\end{gathered}
$$

wasa ${ }^{i \neq j}$ and $s \neq k$ should be; $F-t, X(t), X\left(t-r_{i}\right), \dot{X}\left(t-\tau_{s}\right)$ or $X(t+\theta), \dot{X}(t+\theta)$ consists of a matrix function or functional depending on

$$
\begin{aligned}
& \int_{0}^{r} X(t+\theta) d \xi(\theta), \int_{0}^{\tau} X(t+\theta) d \eta(\theta)-\quad \text { Shows the Stiltiers integral and } \\
& \int_{0}^{\tau} \dot{X}(t+\theta) d \eta(\theta) \neq \dot{X}(t) ; \quad X(t), F, \quad n \text { column of order chi } \\
& X(t)=\left[\begin{array}{l}
x_{1}(t) \\
x_{2}(t) \\
\vdots \\
x_{n}(t)
\end{array}\right], \quad F=\left[\begin{array}{l}
f_{1} \\
f_{2} \\
\vdots \\
f_{n}
\end{array}\right]
\end{aligned}
$$

consists of matrices; $R \ni t$ - optional free parameter; unknown function
$\int_{0}^{\tau} \dot{X}(t+\theta) d \eta(\theta) \neq \dot{X}(t)-\quad$ execution (1.2) of $\dot{X}(t)$ shows that it is solved with respect to.
In some cases, (1.1) or (1.2) ga $n$ nknown ta, is also called the matrix form of the system of differential-functional equations of the first order.

Also $F-$ matrix function, only, $X\left(t-r_{i}\right)$ va $\dot{X}\left(t-\tau_{s}\right)$ or $\int_{0}^{r} X(t+\theta) d \xi(\theta)$ and $\int_{0}^{\tau} \dot{X}(t+\theta) d \eta(\theta)$ is linear with respect to, (1.1) or (1.2) is linearly finitedifferenced is called a differential or differential-functional equation.

Description. 1.2. The equation given in the following form is called a chiorder finite-difference and differential-functional equation, respectively:

$$
\begin{gather*}
x^{(n)}(t)=G\left(t, x(t), x\left(t-r_{1}\right), \ldots, x\left(t-r_{n}\right),\right. \\
\dot{x}\left(t-\tau_{1}\right), \dot{x}\left(t-\tau_{2}\right), \ldots, \dot{x}\left(t-\tau_{m}\right), \\
\left.x^{(n)}\left(t-v_{1}\right), x^{(n)}\left(t-v_{2}\right), \ldots, x^{(n)}\left(t-v_{p}\right)\right) \tag{1.3}
\end{gather*}
$$

and

$$
\begin{equation*}
x^{(n)}(t)=G\left(t, \int_{0}^{r} x(t+\theta) d \xi(\theta), \int_{0}^{\tau} \dot{x}(t+\theta) d \eta(\theta), \ldots, \int_{0}^{v} x^{(n)}(t+\theta) d l(\theta)\right) \tag{1.4}
\end{equation*}
$$

here $\quad v_{p} \in R, v_{p} \neq v_{q}(p=1,2, \ldots, l ; q=1,2, \ldots, l) ; \dot{x}(t)=\frac{d x(t)}{d t}, \quad x^{(n)}(t)=\frac{d^{n} x(t)}{d t^{n}} ; \quad v_{p} \in R$ and $v_{p} \neq 0 ; x(t)-$ scalar, unknown function, $N \ni n-$ consists of the order of the equation; scalar function $t, x(t), x\left(t-r_{i}\right), \dot{x}(t), \dot{x}\left(t-\tau_{s}\right), \ldots, x^{(n-1)}(t), x^{(n)}\left(t-v_{p}\right)$ or $\int_{0}^{r} x(t+\theta) d \xi(\theta), \int_{0}^{\tau} \dot{x}(t+\theta) d \eta(\theta), \ldots, \int_{0}^{v} x^{(n)}(t+\theta) d l(\theta)$ scalar functional depending on ; $\int_{0}^{v} x^{(n)}(t+\theta) d l(\theta) \neq x^{(n)}(t)-$ to be (1.4) ni $X^{(n)}(t)$ ga shows that it is solved with respect to .

Using definition (1.1) and (1.2) and the elementary substitutions used in relation to them, the following statement can be directly confirmed. That is, from (1.1) or (1.2) to (1.3) or (1.4) and vice versa, from (1.3) or (1.4) to (1.1) or (1.2) directly can pass. Therefore, during the study of the specifics of (1.1) or (1.2) and (1.3) or (1.4) finite difference or differential-functional equations, whichever one is suitable for the purpose is taken and studied. From now on, we will dwell more on the considerations that are confirmed in relation to equations (1.1) and (1.3). We consider (1.2) and (1.4) only when necessary.

Description. 1.3. If in the function on the right side of equation (1.1). $\dot{X}\left(t-\tau_{1}\right), \dot{X}\left(t-\tau_{2}\right), \ldots, \dot{X}\left(t-\tau_{m}\right)$ to any of the unknowns $F$ if not related ( does not participate in) or he

$$
\begin{equation*}
\dot{X}(t)=F\left(t, X(t), X\left(t-r_{1}\right), \ldots, X\left(t-r_{n}\right)\right) \tag{1.5}
\end{equation*}
$$

is in it $r_{i}>0(i=1,2, \ldots, n)$ is, then (1.1) is called a differential-functional equation of delayed type (with delayed argument). Also
$\dot{X}\left(t-\tau_{1}\right), \dot{X}\left(t-\tau_{2}\right), \ldots, \dot{X}\left(t-\tau_{m}\right)$ to at least one of $F$ depending on (participating in), $r_{i}>0$ va $\tau_{s}>0 \quad(i=1,2, \ldots, n ; s=1,2, \ldots, m)$ is, then (1.1) is of neutral type, $r_{i}$ and $\tau_{s}$ if the signs of are not clearly indicated, then (1.1) is called a differentialfunctional equation of transitive type.

In some cases, it is possible to switch from one of the equations of delayed, neutral, transient type to another based on some law or rule. But, in general, it is impossible to pass.

Therefore, each of them is studied separately.
(1.1) or (1.2) participating in finite difference differential or differentialfunctional equations studied above ${ }^{r_{i}, \tau_{s}, v_{p}}$ or $\theta$ all of them were assumed as constant numbers.

In practice, they: ${ }^{r_{i}=r_{i}(t), \tau_{s}=\tau_{s}(t), v_{p}=v_{p}(t)}$ yoki $\theta=\theta(t)$
to a free parameter; $r_{i}=r_{i}(t, X(t)), \tau_{s}=\tau_{s}(t, X(t))$,

$$
v_{p}=v_{p}(t, X(t)) \text { or } \theta=\theta(t, X(t))
$$

free parameter, unknown function $-X(t) \mathrm{ga}$;

$$
\begin{gathered}
r_{i}=r_{i}(t, X(t), X(t-a)), \tau_{s}=\tau_{s}(t, X(t), X(t-a)), \\
v_{p}=v_{p}(t, X(t), X(t-a)) \text { yoki } \theta=\theta(t, X(t), X(t-a))
\end{gathered}
$$

free parameters, equations depending on unknown functions and other forms are also studied. Depending on their characteristics, they are divided into types.

Description. 1.4. If (1.1) or (1.2) is a differential or differential-functional equation with a delayed argument

$$
\begin{equation*}
X(t)=\varphi(t) \tag{1.6}
\end{equation*}
$$

funktsiyani ayniyatga aylantirsa, u holda ${ }^{\varphi(t)}$-matritsa funktsiyaga, uning bitta hususiy yechimi, ihtiyoriy o'garmas $n$ chi tartibli kvadrat ${ }^{-C}$ matritsa yordamida hosil qilingan

$$
X(t)=\varphi(t ; C)
$$

funktsiyani ayniyatga aylantirsa, u holda $\varphi(t ; C)$-matritsali funktsiyaga, uning umumiy yechimi deyiladi.

Berilgan, har qanday differentsial-funktsional tenglamaning xususiy yechimlari cheksiz ko'p bo'lishi mumkin. Lekin, uning umumiy yechimi ${ }^{\prime}$ 'zgarmas ${ }^{C}$ ga nisbatan bitta bo’ladi. Ushbularni hisobga olgan holda, (1.1)
yoki (1.2) ning yechimini mavjudligi va yagonaligi haqidagi tushunchalar o'rganiladi. Uning yechimi mavjud bo'lsa, unday yechimlarni topish (aniqlash) usullari keltirib chiqariladi.

1. (1.5) ko'rinishda berilgan, kechikkan argumentli differentsial-funksional tenglama uchun Koshi masalasi quyida shaklda qo'yiladi:
1) (1.5) tenglamaning $\left(t_{0} ; \infty\right)$ da shunday $X(t)=\varphi\left(t, t_{0}\right)$ yechimini topish talab etiladiki, u yechim

$$
\begin{equation*}
\varphi\left(t_{0} ; t_{0}\right)=X\left(t_{0}\right) \tag{1.7}
\end{equation*}
$$

shartni qanoatlantirsin. Bu masala, ayrim hollarda $\left(-\infty ; t_{0}\right)$ yoki $(-\infty ; \infty)$ oraliqlar uchun ham o'rganiladi. Umuman olganda, (1.5) tenglamaning (1.7) shartni qanoatlantiruvchi yechimi yagona emas yoki $\left(t_{0} ; X\left(t_{0}\right)\right)$ nuqtadan o'tuvchi va (1.5) tenglamani qanoatlantiruvchi $\varphi\left(t ; t_{0}\right)$ matritsa funktsiyalar cheksiz ko'pdir. Xususiy holda yoki ${ }^{X(t)-}$ skalyar funktsiyadan iborat.
2) $\left[r ; t_{0}\right]$ ga tegishli bo'lgan $t$ ning barcha qiymatlarida aniqlangan $p(t)$ matritsa funktsiyaga nisbatan, $\left(t_{0} ; \infty\right)$ da (1.5) ning shunday $X(t)=\varphi(t ; p(t))$ yechimini topish talab etiladiki, $u$ yechim

$$
\begin{equation*}
t \in\left[r ; t_{0}\right] \rightarrow \phi(t ; p(t))=p(t) \tag{1.8}
\end{equation*}
$$

shartni qanoatlantirsin.

## Adabiyotlar

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