DIFFERENSIAL TENGLAMALARNI EYLER USULIDA TAQRIBIY YECHISH VA HAQIQIY QIYMAT BILAN SOLISHTIRISH

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Annotatsiya: Oddiy differensial tenglamalar kursidan ma'lumki, agar $y'=f(x,y)$ differensial tenglamada $f(x,y)$ funksiya murakkab bo'lsa, tegishli yechimni topishida taqribiy usullarini qo'llash lozim bo'ladi. Ushbu maqolada hosilaga nisbatan yechilgan differensial tenglamalarni taqribiy hisoblashga oid tushunchalar va misollar ko'rib chiqilgan va analitik yechim bilan solishtirilgan.

Kalit so'zlar: Differensial tenglama, taqribiy yechim, analitik yechim, Koshi masalasi, Eyler usuli, taqribiy qiymat.

APPROXIMATE SOLUTION OF DIFFERENTIAL EQUATIONS USING EULER'S METHOD AND COMPARISON WITH REAL VALUE

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Annotation: It is known from the course of ordinary differential equations that if the function $f(x,y)$ in the differential equation $y'=f(x,y)$ is complex, it is necessary to use approximate methods to find the appropriate solution. In this article, the concepts and examples of the approximate calculation of differential equations solved with respect to the derivative are considered and compared with the analytical solution.

Keywords: Differential equation, approximate solution, analytical solution, Cauchy problem, Euler's method, approximate value.

Differensial tenglamalar kursini o'rganish jarayonida maxsus ko'rinishlarga ega bo'lgan differensial tenglamalarni yechish usullarini o'rganganmiz. Bu usullar juda ko‘p boshqa holatlarni qamrab ololmaydi. Shuning uchun ham tenglama ko‘rinishiga bog‘liq bo‘lmagan universal usullarni qidirishga sabab bo‘ldi. Hisoblash mashinalarining rivojlanishi taqribiy sonli usullarni muvoffaqiyatli qo‘llanilish imkoniyatini yaratdi. Birinchi tartibli differensial tenglamalar uchun Koshi masalasidan boshlaylik.

Aytaylik

$$y' = f(x, y), x_0 \le x \le b \qquad (1)$$

Ko‘rinishdagi differensial tenglamani

$$y(x_0) = y_0 \qquad (2)$$

Boshlang‘ich shartni qanoatlantiruvchi yechimini topish masalasi ya’ni Koshi masalasi berilgan bo’lsin. Umumiy holda Koshi masalasini har doyim ham topish mumkin emas. $f(x, y)$ funksiyaning ma’lum ko‘rinishlaridagina (1) ning umumiy yechimini topish usullari mavjud. Amaliy masalalarda ko‘p hollarda differensial tenglamalarni taqribiy yechish usullaridan foydalaniladi. Bunda yechimning mavjudligi va yagonaligi haqidagi teorema shartlari bajarilgan deb faraz qilinadi. $M_0(x_0; y_0)$ nuqta atrofida $f(x, y)$ funksiya x bo‘yicha uzluksiz, y bo‘yicha esa Lipshis shartini qanoatlantirsin.

Eyler usuli: (1)-(2) Koshi masalasi yechimi $y(x)$ ni x_0 nuqta atrofida Teylor qatoriga yoyamiz:

$$y(x) = y_0 + (x - x_0) y'(x_0) + \frac{(x - x_0)^2}{2!} y''(x_0) + \frac{(x - x_0)^3}{3!} y'''(x_0) + \ldots \qquad (3)$$

x_0 nuqtaning kichik atrofida Teylor qatorining birinchi ikkita hadini olib, qolgan hadlarini tashlab yuboramiz, natijada quyidagicha taqribiy formulaga kelamiz

$$y(x) \approx y_0 + (x - x_0) y' \qquad (4)$$

agar y' ning (1) formuladagi ko‘rinishidan foydalansak, u holda (4) formulani quyidagi ko‘rinishda yozish mumkin:

$$y(x) \approx y_0 + (x - x_0) f(x_0; y_0) \qquad (5)$$

(5) formulani $x_0 \le x \le b$ oraliqqa umumlashtirish uchun, ushbu oraliqni n ta bo‘lakka bo‘lamiz. Bo‘laklash qadami:

$$h=\frac{b-x_0}{n};x_i=x_0+ih,i=0,1,2,3,\ldots,n$$

Masala yechimini x_i nuqtalarda jadval ko‘rinishida topishni maqsad qilib qo‘yamiz. Taqribiy $y(x_i)$ ning qiymatlarini (5) formula bo‘yicha topamiz:

$$y_{i+1}\approx y_i+h\cdot f(x_i,y_i)i=0,1,2,3,\ldots,n-1 \qquad (6)$$

bunda $y_{i+1}=y(x_{i+1}),y_i=y(x_i)$. Ushbu formulaga **Eyler usuli** deyiladi. Eyler usuli universal usul bo‘lib, $f(x,y)$ ning ko‘rinishiga bog‘liq emas, lekin xatolik nisbatan katta. Har qadamdagi xatolik $O(h^2)$ tartibida bo‘lib, bu xatolik qadamma-qadam ortib borib, b nuqtaga yetib borguncha xatolik $O(h)$ gacha ortishi mumkin. Koordinatalar tekisligida $(x_0,y_0);(x_1,y_1);\ldots,(x_n,y_n)$ nuqtalarni to‘g‘ri chiziq kesmalari bilan tutashtirishdan hosil bo‘lgan siniq chiziq integral egri chiziqning grafigi bo‘ladi.

1-misol. $y'=x^2+y=f(x,y);y(0)=0,3$ boshlang‘ich shartni qanoatlantiruvchi yechimning $x=0,3$ dagi taqribiy qiymatini Eyler usulida toping. Bu yerda $h=0,1$.

Yechish. Eyler usuli:

$$x_0=1;x_i+i\cdot h=1+i\cdot 0.2;y_0=y(1)=2$$

$$y_1=y_0+h*f(x_0,y_0)=2+0.2*(1^2+2)=2.6$$

$$y_2=y_1+h*f(x_1,y_1)=2.6+0.2*(1.2^2+2.6)=3.408$$

$$y_3=y_2+h*f(x_2,y_2)=3.408+0.2*(1.4^2+3.408)=4.4816$$

$$y_4=y_3+h*f(x_3,y_3)=4.4816+0.2*(1.6^2+4.4816)=5.890$$

$$y_5=y_4+h*f(x_4,y_4)=5.890+0.2*(1.8^2+5.890)=7.716$$

Shunday qilib differensial tenglamaning Eyler usulida taqribiy yechimi quydagicha jadval ko’rinishda bo’ladi:

x_i	1	1.2	1.4	1.6	1.8	2.0
y_i	2	2.6	3.408	4.4816	5.890	7.716

Aniqlik uchun differensial tenglamaning analitik usulda topilgan yechimini ham keltiramiz:

Yechish:

$$y'=x^2+y,y(1)=2$$

$y'-y=x^2,e^{\int -dx}=e^{-x}$

$$y' e^{-x} - e^{-x} y = x^2 e^{-x} \Rightarrow \left(y e^{-x}\right)' = x^2 e^{-x} dx$$

$$y e^{-x} = \int x^2 e^{-x} dx = \begin{vmatrix} u = x^2 & dv = e^{-x} dx \\ du = 2x dx & v = -e^{-x} \end{vmatrix} = -x^2 e^{-x} + 2\int x e^{-x} dx = ¿$$

$$¿ - x^2 e^{-x} + \int x e^{-x} dx = \begin{vmatrix} u = x & dv = e^{-x} dx \\ du = dx & v = -e^{-x} \end{vmatrix}$$

$$¿ - x^2 e^{-x} - 2x e^{-x} - 2e^{-x} + C \Rightarrow$$

$$\Rightarrow y = C e^{x} - x^2 - 2x - 2$$

Endi $y(1)=2$ qiymatga ko‘ra

$$2 = Ce - 1 - 2 - 2 \Rightarrow C = \frac{7}{e}$$

$$y = -x^2 - 2x + 7e^{x-1} - 2$$

Analitik usulda yechganda differensial tenglama yechimi

$$y(x) = -x^2 - 2x + 7e^{x-1} - 2$$

ko’rinishda bo’lib, biz topgan taqribiy qiymatlar asl yechim qiymatlariga solishtirganda mos qiymatlar juda yaqin ekanligini ko‘rishimiz mumkin.

Bundan ko‘rinib turibdiki, Eyler usuli orqali topilgan taqribiy yechim va analitik usulda topilgan yechimning mos qimatlari bir-biriga ancha yaqin bo‘lishini ko‘rishimiz mumkin.

Foydalanilga adabiyotlar ro‘yxati.

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