MATHEMATICAL MODEL AND NUMERICAL SIMULATION ON THE DIFFUSION OF ALCOHOL IN THE HUMAN BODY

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Abstract:

This paper mainly studies the mathematical model on the diffusion of alcohol in the human body after drinking, and the mathematical model on the diffusion of alcohol in the human boday is established under two hypotheses. At first, the differential dynamic model on the diffusion of alcohol in the human body is established after drinking in a short period, and then a mathematics model on the diffusion of alcohol in the human body is established after dringking at an even speed. The author also proves the application of the model by combining the numerical simulation technology with the practical cases.

Considering the actual process of drinking, as the alcohol enters into the body at neither an instant speed nor an even speed, based on the first two differential equation models, in the latter part the author establishes the impulsive differential dynamic system model, which has the diffuse and impulsive effect for the alcohol in the human body. This model is closer to the actual process. The author simulates the changing process of alcohol concentration with the aid of numerical simulation.

Finally, the author summarizes the results and prospects the direction of future research.

Key words:

Alcohol spread; differential equations; impulsive differential equations; the numerical simulation

1 The basic assumptions and symbols

1.1 The basic assumptions

1. Human body fluids (such as blood, lymph, tissue fluid) account for 65% to 70% of the total weight of the human body, but only 7% of the fluids are blood. The percentage of the alcohol (incouding the drug) in the blood is the same as that in the fluids, so we assume in this paper that the content of alcohol in the blood at any time equals that in body fluids.

2. The transfer of alcohol is divided into four stages: alcohol - stomach- body fluids-liver- body;

3. All alcohol entering into the stomach is diffused into the body fluid; the alcohol in the body fluids goes into the liver; the alcohol in the liver is totally discharged out of the body through the liver oxidative decomposition.

4. The transfering rate of alcohol in the body and discharge rate is proportional to the alcohol content of the body.

1.2 Symbols that

$x_1(t)$: The amount of alcohol in the stomach at time t;

$x_2(t)$: The amount of alcohol in fluids at time t

$x_3(t)$: The amount of alcohol in the liver at time t;

$f_1(t)$: The transfering rate of alcohol content from the stomach to fluid;

$f_2(t)$: The transfering rate of alcohol content from the body fluid to the liver;

k_1 :The ransfering rate coefficient of alcohol from the stomach into the fluid;

k_2: The transfering rate coefficient of alcohol from the fluid into the liver;

k_3: The transfering rate coefficient of alcohol decomposition by the liver and to the discharging;

V_1: The volume of fluids;

V_2: The volume of the liver;

$c_1(t)$:The alcohol content in body fluid (blood);

$c_2(t)$:The alcohol content in liver;

D_0 :Amount of alcohol into the stomach;

t_1^*: The time for alcohol contents reach peak under the condition of drinking for a relatively short period of time.

t_2^*: The time for alcohol contents reach peak under the condition of drinking for a longer period.

2 The mathematical model on the diffusion of alcohol in the human body after drinking alcohol in a short period of time.

2.1 The establishment of the model

Assume that under the condition of drinking for a relatively short period of time, alcohol enters the stomach instantly. Ignore the time of drinking alcohol, for example, drinking a bottle of beer at one time.

According to the assumptions, $x_2(t)$ satisfy the differential equation is:

$$\frac{dx_2(t)}{dt} = -k_2 x_2(t) + f_1(t) \tag{1}$$

$x_2(t)$ With the fluid in the volume V_1 and the relation between the blood alcohol level $c_1(t)$ is:

$$x_2(t) = V_1 c_1(t) \tag{2}$$

$x_1(t)$ Satisfy the differential equations

$$\begin{cases} \frac{dx_1(t)}{dt} = -k_1 x_1(t) \\ x_1(0) = D_0 \end{cases} \tag{3}$$

And according to the hypothesis

$$f_1(t) = k_1 x_1(t) \tag{4}$$

Solution (3) and (4)

$$f_1(t) = k_1 D_0 e^{-k_1 t} \tag{5}$$

(5) into (1)

$$\begin{cases} \frac{dx_2(t)}{dt} = -k_2 x_2(t) + k_1 D_0 e^{-k_1 t} \\ x_2(0) = 0 \end{cases} \tag{6}$$

Of solution

$$x_2(t) = \frac{k_1 D_0}{(k_2 - k_1)}(e^{-k_1 t} - e^{-k_2 t}) \tag{7}$$

(2) into (7), too

$$c_1(t) = \frac{k_1 D_0}{V_1(k_2 - k_1)}(e^{-k_1 t} - e^{-k_2 t}) \tag{8}$$

order $c_1'(t) = 0$, get

$$t_1^* = \frac{\ln k_2 - \ln k_1}{k_2 - k_1} \tag{9}$$

So in t_1^* time $c_1(t)$ reach maximum.

$x_3(t)$ satisfy the differential equations

$$\frac{dx_3(t)}{dt} = -k_3 x_3(t) + f_2(t) \tag{10}$$

$$f_2(t) = k_2 x_2(t) \tag{11}$$

(7) into (11)

$$\begin{cases} \dfrac{dx_3(t)}{dt} = -k_3 x_3(t) + \dfrac{k_1 k_2 D_0}{k_2 - k_1}(e^{-k_1 t} - e^{-k_2 t}) \\ x_3(0) = 0 \end{cases} \tag{12}$$

Solution of

$$x_3(t) = k_1 k_2 D_0 \left[\frac{e^{-k_1 t}}{(k_2 - k_1)(k_3 - k_1)} + \frac{e^{-k_2 t}}{(k_1 - k_2)(k_3 - k_2)} + \frac{e^{-k_3 t}}{(k_1 - k_3)(k_2 - k_3)} \right] \tag{13}$$

2.2 The numerical simulation

One person with a weight of about 70 kg drinks two bottles of beer in a short period of time and then measure the amout (mg/ml) of alcohol in his blood at intervals. The data is as follows,

Time（hour）	0.25	0.5	0.75	1	1.5	2	2.5	3	3.5	4	4.5	5
Alcohol content	30	68	75	82	82	77	68	68	58	51	50	41
Time（hour）	6	7	8	9	10	11	12	13	14	15	16	
Alcohol content	38	35	28	25	18	15	12	10	7	7	4	

According to the given data, use Matlab software to data fitting (8), and the type of undetermined coefficients for approximate solution can be obtained

$$k_1 = 1.9392，\ k_2 = 0.1901，\ \frac{k_1 D_0}{V_1(k_2 - k_1)} = -117.0571 \tag{14}$$

Fitting the image is shown in figure 1:

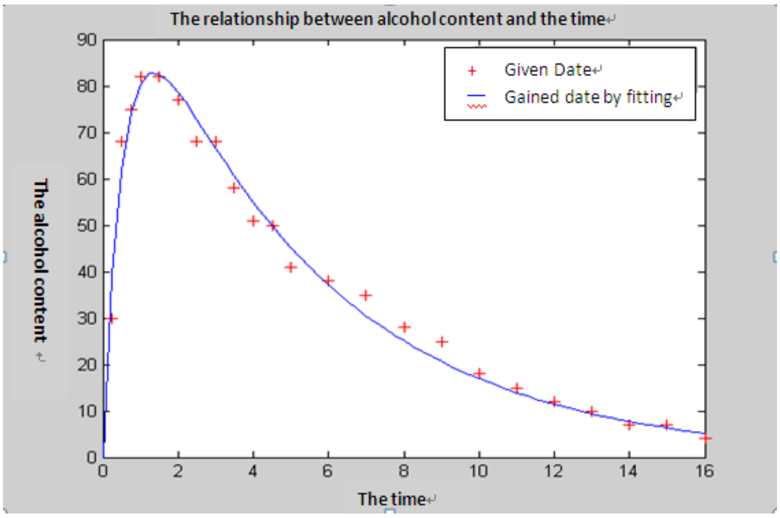

Figure 1: The relationship diagram between the time and the alcohol content in the fluid after fast drinking

Figure1 shows that the fitting result is well and it well reflects the changing rule of alcohol content in human blood and they are in conformity with the actual changes, which also proves that established model meets the basic requirements. (11) into (9), get $t_1^* = \dfrac{\ln(1.9392) - \ln(0.1901)}{1.9392 - 0.1901} \approx 1.33$ available hours, which means that alcohol content in the blood will reach the peak after1.3 hours in the case of fast drinking.

3 The mathematical model on the diffusion of alcohol in the human body after drinking alcohol uniformly

3.1 The establishment of the model

Assuming that drinking is at an even speed and is finished in a long time, the time of drinking is t_0 ,

When $t \le t_0$, $x_1(t)$ satisfy the differential equations

$$\begin{cases} \dfrac{dx_1(t)}{dt} = -k_1 x_1(t) + \dfrac{D_0}{t_0} \\ x_1(0) = 0 \end{cases} \tag{15}$$

Solution of

$$x_1(t) = \frac{D_0}{t_0 k_1}(1 - e^{-k_1 t}) \tag{16}$$

$x_2(t)$ satisfy the differential equations

$$\begin{cases} \dfrac{dx_2(t)}{dt} = -k_2x_2(t) + k_1x_1(t) \\ x_2(0) = 0 \end{cases} \tag{17}$$

Solution of

$$x_2(t) = \frac{D_0k_1}{t_0k_2(k_2-k_1)}\left(e^{-k_2t} - \frac{k_2}{k_1}e^{-k_1t} + \frac{k_2-k_1}{k_1}\right) \tag{18}$$

when $t \geq t_0$, $x_1(t)$ satisfy the differential equations

$$\begin{cases} \dfrac{dx_1(t)}{dt} = -k_1x_1(t) \\ x_1(t_0) = \dfrac{D_0}{t_0k_1}(1-e^{-k_1t_0}) \end{cases} \tag{19}$$

Solution of

$$x_1(t) = \frac{D_0}{t_0k_1}(e^{k_1t_0}-1)e^{-k_1t} \tag{20}$$

$x_2(t)$ satisfy the differential equations

$$\begin{cases} \dfrac{dx_2(t)}{dt} = -k_2x_2(t) + k_1x_1(t) \\ x_2(t_0) = \dfrac{D_0k_1}{t_0k_2(k_2-k_1)}\left(e^{-k_2t_0} - \dfrac{k_2}{k_1}e^{-k_1t_0} + \dfrac{k_2-k_1}{k_1}\right) \end{cases} \tag{21}$$

Solution of

$$x_2(t) = \frac{D_0k_1(1-e^{k_2t_0})}{t_0k_2(k_2-k_1)}\left[e^{-k_2t} - \frac{k_2(e^{k_1t_0}-1)}{k_1(e^{k_2t_0}-1)}e^{-k_1t}\right] \tag{22}$$

So, when drinking for a long time, $c_1(t)$ meet the relation

$$\begin{cases} c_1(t) = \dfrac{k_1D_0}{V_1t_0k_2(k_2-k_1)}\left(e^{-k_2t} - \dfrac{k_2}{k_1}e^{-k_1t} + \dfrac{k_2-k_1}{k_1}\right), & t \leq t_0 \\ c_1(t) = \dfrac{D_0k_1(1-e^{k_2t_0})}{t_0V_1k_2(k_2-k_1)}\left[e^{-k_2t} - \dfrac{k_2(e^{k_1t_0}-1)}{k_1(e^{k_2t_0}-1)}e^{-k_1t}\right], & t \geq t_0 \end{cases} \tag{23}$$

when $t \leq t_0$, $c_1'(t) > 0$, $c_1(t)$ Monotone increasing

when $t \geq t_0$, order $c_1'(t)=0$,

$$t_2^* = \frac{\ln \frac{e^{k_1 t_0} - 1}{e^{k_2 t_0} - 1}}{k_1 - k_2} \tag{24}$$

so $c_1(t)$ in t_2^* time reach maximum

when $t \le t_0$, $x_3(t)$ satisfy the differential equations

$$\begin{cases} \frac{dx_3(t)}{dt} = -k_3 x_3(t) + k_2 x_2(t) \\ x_3(0) = 0 \end{cases} \tag{25}$$

（18）into（25）

$$x_3(t) = \frac{k_1 D_0}{t_0}\left[\frac{e^{-k_2 t}}{(k_2-k_1)(k_3-k_2)} - \frac{k_2 e^{-k_1 t}}{k_1(k_2-k_1)(k_3-k_1)} - \frac{k_2 e^{-k_3 t}}{k_3(k_3-k_1)(k_3-k_2)} + \frac{k_2-k_1}{k_3 k_1}\right] \tag{26}$$

when $t \ge t_0$, $x_3(t)$ satisfy the differential equations

$$\begin{cases} \frac{dx_3(t)}{dt} = -k_3 x_3(t) + k_2 x_2(t) \\ x_3(t_0) = \frac{k_1 D_0}{t_0}\left[\frac{e^{-k_2 t_0}}{(k_2-k_1)(k_3-k_2)} - \frac{k_2 e^{-k_1 t_0}}{k_1(k_2-k_1)(k_3-k_1)} - \frac{k_2 e^{-k_3 t_0}}{k_3(k_3-k_1)(k_3-k_2)} + \frac{k_2-k_1}{k_3 k_1}\right] \end{cases} \tag{27}$$

（22）into（27）

$$x_3(t) = \frac{k_1 D_0 (1-e^{k_2 t_0})}{t_0(k_2-k_1)}\left[\frac{e^{-k_2 t}}{k_3-k_2} - \frac{k_2(e^{k_1 t_0}-1)e^{-k_1 t}}{k_1(k_3-k_1)(e^{k_2 t_0}-1)} + Me^{-k_3 t}\right] \tag{28}$$

$$M = D_0\left[\frac{k_1}{t_0(k_2-k_1)(k_3-k_2)} - \frac{k_2}{t_0(k_2-k_1)(k_3-k_1)} + \frac{k_2-k_1}{t_0 k_3}\right]e^{k_3 t_0} - \frac{D_0 k_1 k_2}{t_0 k_3(k_3-k_1)(k_3-k_2)}$$

3.2 The numerical simulation

(14) of the data into (23), and $t_0=2$, using Matlab software to mapping (23), it is shown in Figure 2:

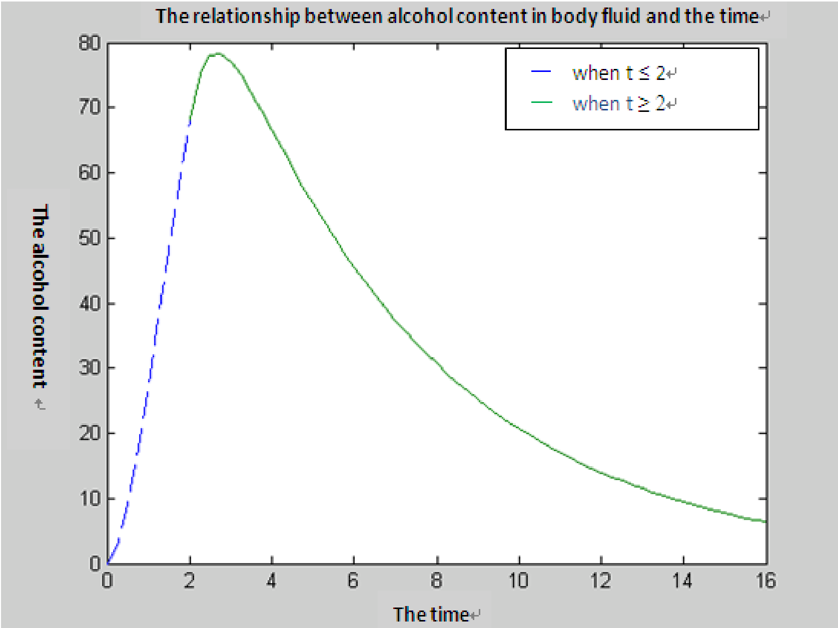

Figure 2: The relationship between time and the alcohol content in the fluid after drinking uniformly.

(14) of the data into (24), and t_0=0 the hours available

$$t_2^* = \frac{\ln \dfrac{e^{2\times 1.9392}-1}{e^{2\times 0.1901}-1}}{1.9392-0.1901} \approx 2.65 \text{ hours}$$

It shows that alcohol content in the blood will peak after 2.7 hours under the condition of uniform drinking for a long time. According to the results, alcohol content in the bloodstream can reach maximum in a short period of time when drinking fast, so a quick drink is more easily drunk than slow drink .Therefore, it is not suggest to drink fast.

4 A mathematical model with the diffuse and impulsive effect for the alcohol in the human body

4.1 The establishment of the model

In general, alcohol is not into the stomach at an even speed when people drink, but at an instant speed. For example, the amount of alcohol in the stomach can suddenly increase when we put our cheers; it is considered as the process of pulse phenomena, its mathematics model are as follows:

$$\begin{cases} \dfrac{dx_1(t)}{dt} = -k_1 x_1(t) & t \neq t_k \\ \Delta x_1(t) = P & t = t_k \\ x_1(t_0) = N_0 \end{cases} \tag{29}$$

P is said the amount of alcohol into the stomach with a quick drink in t_k the moment, N_0 is said the amount of alcohol in the stomach.

When in $t \in (t_0, t_1]$

at the t_0 initial time

$$x_1(t) = N_0 e^{-k_1(t-t_0)} \tag{30}$$

so $x_1(t_1) = N_0 e^{-k_1(t_1-t_0)}$ （31）

$$x_1(t_1^+) = N_0 e^{-k_1(t_1-t_0)} + P \tag{32}$$

So in $t \in (t_1, t_2]$,

$$x_1(t) = x_1(t_1^+) e^{-k_1(t-t_1)} \tag{33}$$

$$x_1(t_2) = x_1(t_1^+) e^{-k_1(t_2-t_1)} = \left[N_0 e^{-k_1(t_1-t_0)} + P\right] e^{-k_1(t_2-t_1)} \tag{34}$$

when $t \in (t_n, t_{n+1}]$, the solution of impulsive differential system is

$$x_1(t) = N_0 e^{-k_1(t-t_0)} + \sum_{i=1}^{n} \left[e^{-k_1(t-t_i)}\right] P \tag{35}$$

Suppose every λ minute to drink a glass of wine,

so $\lambda = t_{i+1} - t_i$, （35） turns

$$x_1(t) = N_0 e^{-k_1(t-t_0)} + P\left[e^{-k_1(t-t_1)} + e^{-k_1(t-t_2)} + \cdots + e^{-k_1(t-t_n)}\right]$$

$$= N_0 e^{-k_1(t-t_0)} + P \frac{e^{k_1 t_1}(1-e^{n\lambda k_1})}{1-e^{\lambda k_1}} e^{-k_1 t} \tag{36}$$

if $N_0 = P$, so （35） turns

$$x_1(t) = \sum_{i=0}^{n} \left[e^{-k_1(t-t_i)}\right] P = P \frac{e^{k_1 t_0}\left[1-e^{(n+1)\lambda k_1}\right]}{1-e^{\lambda k_1}} e^{-k_1 t} \tag{37}$$

As assumed that transfer of alcohol in the human body is divided into four processes: alcohol - stomach-body fluids-liver- body, so the mathematical model with the diffuse and impulsive effect for the alcohol in the human body can be set up as follows,

$$\begin{cases} \dfrac{dx_1(t)}{dt} = -k_1 x_1(t) \qquad t \neq t_i \\ \dfrac{dx_2(t)}{dt} = -k_2 x_2(t) + k_1 x_1(t-\tau_1) \\ \dfrac{dx_3(t)}{dt} = -k_3 x_3(t) + k_2 x_2(t-\tau_2) \\ x_1(t_0) = P, x_2(t_0) = 0, x_3(t_0) = 0 \\ \Delta x_1(t) = P, \Delta x_2(t) = 0, \Delta x_3(t) = 0, \qquad t = t_i, i = 1, 2, \cdots, N \end{cases} \tag{38}$$

System (38) supposes that someone finishes every drink at one time, ie, the alcohol enters into the stomach instantly and drinks one for every λ hours. N is for the number of the drink, $\lambda = t_{n+1} - t_n$ τ_1 is for the delaly caused for diffusion of alcohol from the stomach to the body fluid, τ_2 is for the delaly produced for diffusion of alcohol from the fluid to the liver, when $t \in (t_n, t_{n+1}]$ the solution for the system (38) is

$$\begin{cases} x_1(t) = P\dfrac{e^{k_1 t_0}\left[1-e^{(n+1)\lambda k_1}\right]}{1-e^{\lambda k_1}} e^{-k_1 t} \\ x_2(t) = \dfrac{Pk_1 e^{k_1\tau_1}\left[1-e^{(n+1)\lambda k_1}\right]}{(k_2-k_1)(1-e^{\lambda k_1})}\left[e^{-k_1(t-t_0)} - e^{-k_2(t-t_0)}\right] \\ x_3(t) = \dfrac{Pk_1 k_2 e^{k_1\tau_1}\left[1-e^{(n+1)\lambda k_1}\right]}{(k_2-k_1)(1-e^{\lambda k_1})}\left[\dfrac{e^{-k_1(t-\tau_2-t_0)}}{k_3-k_1} - \dfrac{e^{-k_2(t-\tau_2-t_0)}}{k_3-k_2} - \left(\dfrac{e^{k_1\tau_2}}{k_3-k_1} - \dfrac{e^{k_2\tau_2}}{k_3-k_2}\right)e^{-k_3(t-t_0)}\right] \end{cases} \tag{39}$$

4.2 **The numerical simulation**

Suppose someone starts drinking alcohol at initial time $t_0 = 0$, and the amount of alcohol in the stomach is $x_1(0) = 10$ ml at initial time, amount for every glass of alcohol is10 ml, a total of ten cups of wine need drinking and need drinking at the fastest speed for every drink, it is in (39)

$t_0 = 0, P = 10, n = 10, k_1 = 1.9392, k_2 = 0.1901$

1. If drink a glass of wine for every 3 minutes, or $\lambda = 0.05$ hours, the amount of alcohol in the stomach changes over time, which is shown in figure 3, alcohol content in body fluids changes over time is shown in figure 4.

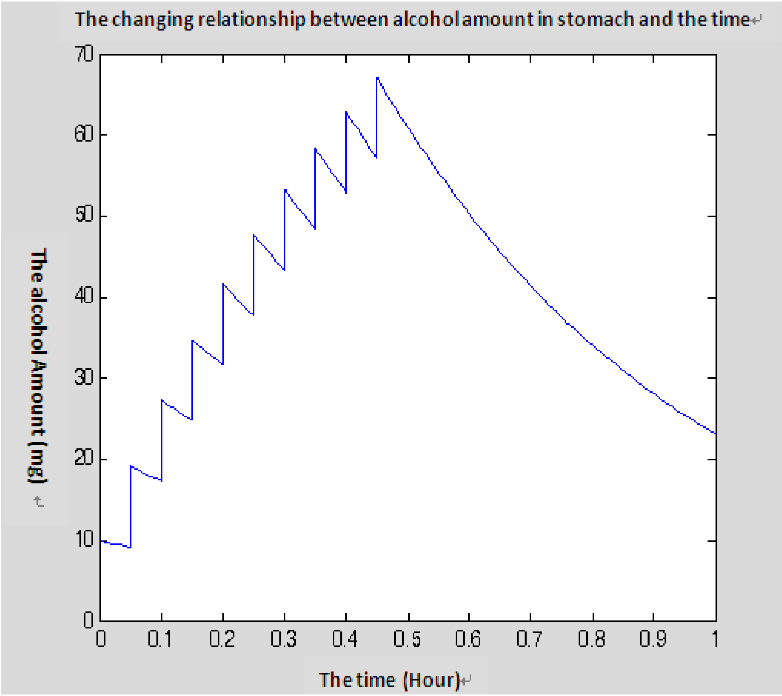

Figure 3: The changing relationship between alcohol amount in stomach and the time when drinking for every 3 minutes.

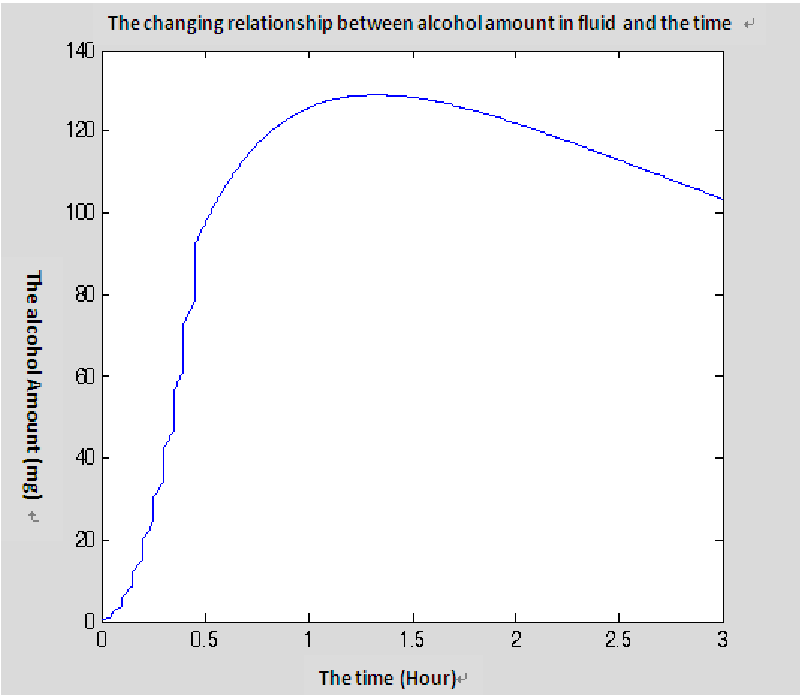

Figure 4: The changing relationship between alcohol amount in body fluid and the time when drinking for every 3minutes.

2. If drink a glass of wine for every 20 minutes, or $\lambda = \frac{1}{3}$ hours, the amount of alcohol in the stomach changes over time is shown in figure 5.

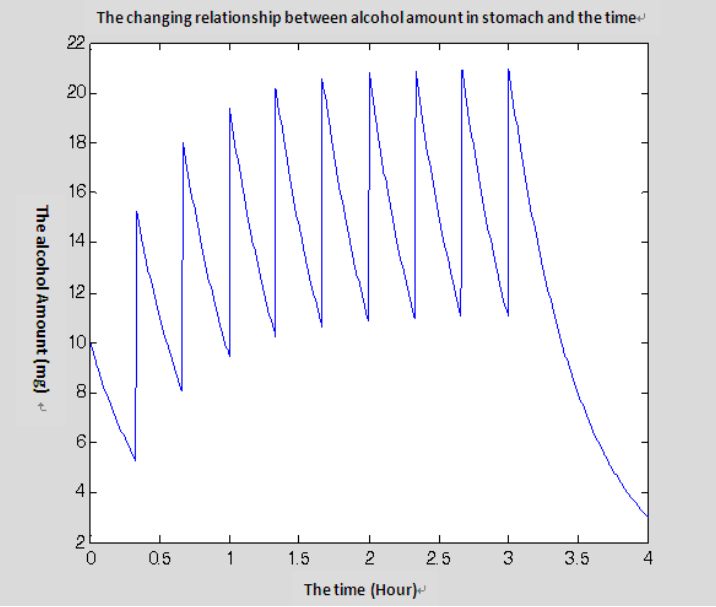

Figure 5: The changing relationship between alcohol amount in stomach and the time when drinking for every 20 minutes.

3. If drink a cup of wine for every 60 minutes or $\lambda = 1$ hours, the amount of alcohol in the stomach changes over time is shown in figure 6.

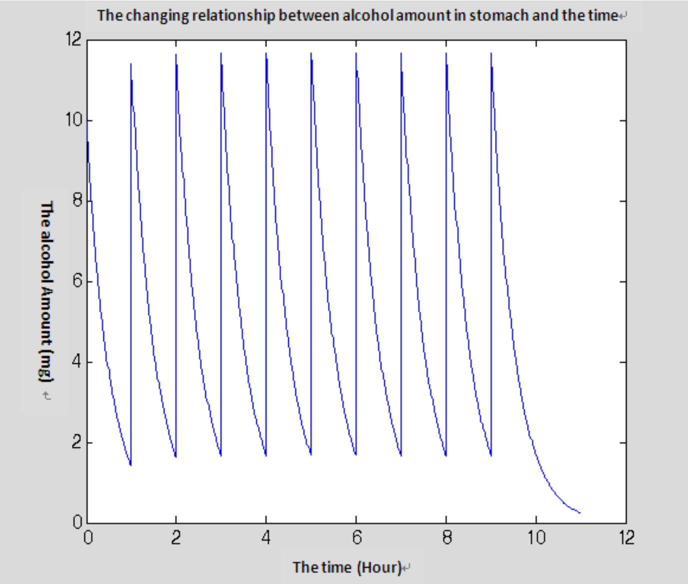

Figure 6: The changing relationship between alcohol amount in stomach and the time when drinking for every 60 minutes.

In general, especially when someone drink white sprite, it is not possible to finish it at one gulp and it will last longer. The alcohol is not likely to be evenly into the stomach, so the model is closer to actual situation than the previous two models.

This model describes after drinking, in the process of alchohol is transferred to different organs, the alcohol is constantly obsorbed, distributed, metabolized and finally discharged out of body. Therefore, research about alcohol's absorption, distribution and dynamic process in the human body has important guiding function and practical usage for the treatment of mental disorder caused by drinking too much and movement disorders, respiratory dysfunction, alcoholic myocarditis, hypertension, fatty liver, liver cirrhosis, alcohol and fetal alcohol syndrome and so on.

5 Conclusion

This article mainly studies the mathematical model on the alcohol diffusion in the human body. Under the basic assumptions that the transferring process of alchol in the human body is divided into four stages (alcohol - stomach- body fluids-body), the mathematical model is established and numerical simulation is verificated on the diffusion process when drinking in different ways.

First, a mathematics model on the alcohol diffusion in the human body is established after drinking in a short period of time, and the research results are verified by the use of numerical simulation technology.

And then a mathematics model on the alcohol diffusion in the human body is established after drinking uniformly, the research results are verified by the use of numerical simulation technology.

Finally on the basis of the first two chapters, a mathematical model with the diffuse and impulsive effect for the alcohol in the human body is established and the research results are verified by using numerical simulation technology.

As there is not enough information, the detailed alcohol diffusion process in the human body is not understood fully and only a simple hypothesis is made, which causes the difference, such as the breathing process of drinking and aftern drinking, the process for alcohol absorbed by different organs when it diffuses in the body. In addition, there is no enough data, so most of things can not be estimated such as the transfer rate coefficient for the decomposition of alcohol by the liver, the time delay produced when diffusing from the stomach to the body fluids and the delay casued by diffusing from body fluids to the liver.

This article assumes that k_1, which is transfer rate coefficient for alcohol from the stomach into the fluid, is constant, but in the actual situation, k_1 is influenced by many factors, such as:

1. The alcohol with low concentration diffuses slow in the stomach, and if the concentraton is too high, the gastric mucosal will be damaged, diffusion is slow;
2. Alcohol diffuses quickly in an empty stomach, slowly in a full stomach;
3. Alcohol diffuses slowly when eating with milk, fat and sweets;
4. The diffusion of beer is slower than that of spirit.

So assuming k_1 for variables conforms more to the actual situation.

Every time when people drinking, the amount of alcohol and the time intervals are ramdom, so establishing a mathematical model with random pulse effect conforms more to the actual situation.

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