# BIRINCHI VA IKKINCHI TUR SIRT INTEGRALLARI.

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**ANNOTATSIYA:** Ushbu maqolada matematikaning eng qiziq mavzularidan biri bo'lgan Birinchi va ikkinchi tur sirt integrallari haqida ma'lumotlar berib o'tildi va mavjud muanmolarga ilmiy yondashildi hamda muanmolarni hal etish uchun tegishli tavsiyalar berib o'tildi.

**KALIT SO'ZLAR:** Birinchi tur sirt integrallari, Ikkinchi tur sirt integrallari.

## First and second round surface integrals.

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**ABSTRACT:** This article provides information about the first and second types of surface integrals, which are one of the most interesting topics in mathematics, and provides a scientific approach to existing problems and provides relevant recommendations for solving problems.

**KEYWORDS:** First type surface integrals, Second type surface integrals.

## 1. Birinchi tur sirt integrallari.

$\sigma -$ birorta silliq sirt va f(x,y,z )= f(M ) funkція $\sigma$ sirtda uzluksiz bo'lsin; $\Delta\sigma_1, \Delta\sigma_2, \dots \Delta\sigma_n$ lar $\sigma$ sirtning elementar sirtlarga bo'linishi bo'lib, ularning yuzlarini ham shu simvollar bilan belgilaylik; har qaysi elementar sirtda ixtiyoriy $M_i(x_i, y_i, z_i)$ nuqta tanlaymiz va ushbu $\sum_{i=1}^{n} f(x_i, y_i, z_i)\,\Delta\sigma_i$ integral yig'indini tuzamiz.

Elementar sirtlarning diametrining eng kattasi nolga intilganda integral yig'indi intiladigan limit birinchi tur sirt integrali (yoki sirt yuzi bo'yicha integral) deyiladi: $\iint_\sigma f\,(x, y\; z)d\sigma = \lim_{maxdiam\Delta\sigma_{i\to 0}} \sum_{i=1}^{n} f(x_i, y_i, z_i)\,\Delta\sigma_i$

*yoki*

$$\iint_\sigma f\,(M)d\sigma = \lim_{maxdiam\Delta\sigma_{i\to 0}} \sum_{i=1}^{n} f(x_i, y_i, z_i)\,\Delta\sigma_i$$

Sirt integralining qiymati $\sigma$ sirtning qaysi tomoni tanlanishiga bog'liq emas.

Aniq integralning barcha xossalari birinchi tur sirt integrallari uchun o’rinlidir. Agar $\sigma$ sirtning Oxy tekislikka proyeksiyasi $\sigma_{xy}$ bir qiymatli bo’lsa, ya’ni Oz o’qqa parallel har qanday to’g’ri chiziq $\sigma$ sirtni faqat bitta nuqtada kessa, mos birinchi tur sirt integralni hisoblashni ushbu formula orqali ikki o’lchovli integralni hisoblashga keltirish mumkin:

$$\iint_{\sigma} f\,(x,y,z)d\sigma = \iint_{\sigma_{xy}} f(x,y,z(x,y))\sqrt{1+z_x'^2 + z_y'^2}\ dxdy,$$

bu yerda z = z(x, u) — $\sigma$ sirtning tenglamasi. Ravshanki,

$$\iint_{\sigma} d\sigma = S,$$

bu yerda S — $\sigma$ sirtning yuzi, bu yerda

$$\iint_{\sigma} f\,(x,y,z)d\sigma = M,$$

bu yerda

M — $\sigma$sirtning massasi, f(x, u, z) = u — $\sigma$ sirtning sirtiy zichligi.

1- m i s o l. $\iint_{\sigma}(x^2+y^2)d\sigma$integralni hisoblang, bu yerda

$\sigma - x^2+y^2 = z^2$ конус сиртининг $z=0$ ва $z=1$ tekisliklar orasidagi qismi.

Yechish. Berilgan $\sigma$ sirt tenglamasidan uning qaralayotgan qismi uchun z=$\sqrt{x^2+y^2}$ ekanini ko’ramiz :

$z_x = \frac{x}{\sqrt{x^2+y^2}}$, $z_{y=}\frac{y}{\sqrt{x^2+y^2}}$

Demak,

$$\iint_{\sigma}(x^{2+}y^2)d\sigma = \iint_{x_y}(x^2+y^2)\sqrt{1+\frac{x^2}{x^2+y^2}+\frac{y^2}{x^2+y^2}}+dxdy$$

$$\sqrt{2}\iint_{\sigma_{xy}}(x^2+y^2)dxdy$$

Ikki o’lchovli integralning integrallash sohasi $\sigma_{xy}\, x^2+y^2 \le 1$doiradan iborat (konus sirtning Oxu tekislikka proektsiyasi).

Ikki o’lchovli integralda kutb koordinatalariga o’tamiz:

$$\sqrt{2}\iint_{\sigma_{xy}}(x^2+y^2)dxdy = \sqrt{2}\iint_{\sigma_{xy}} r^3 dr d\varphi = \sqrt{2}\int_0^{2\pi} d\varphi \int_0^1 r^3\, d\varphi =$$

$$\sqrt{2}\int_0^{2\pi}\left(\frac{1}{4}r^4|_0^1\right)d\varphi = \frac{\sqrt{2}}{4}\int_0^{2\pi} d\varphi = \frac{\pi\sqrt{2}}{2}$$

$\sigma$ silliq sirtning xar bir nuqtasidan $\vec{n}$ normal vektori o’tkazilgan tomoni musbat, boshqa tomoni (agar u mavjud bo’lsa) esa manfiy tomon deyiladi.

Xususan, agar $\sigma$ sirt yopiq bo'lsa va Q fazoning biror sohasini chegaralasa, u holda sirtning musbat yoki tashqi tomoni deb uning normal vektorlar Q sohadan yo'nalgan tomoni, manfiy yoki ichki tomoni deb uning normal vektorlari Q sohaga yo'nalgan tomoni aytiladi. Musbat (tashqi) va manfiy (ichki) tomonlari mavjud bo'lgan sirtlar ikki tomonlama sirtlar deyiladi. Ular uchun quyidagi xossa o'rinlidir. Agar $\vec{n}$ normal vektorning asosini bunday sirtda yotuvchi istalgan yopiq L kontur bo'ylab uzluksiz ko'chirilsa, dastlabki nuqtaga kaytganda $\vec{n}$ ning yo'nalishi dastlabki yo'nalish bilan bir xil bo'ladi.

Bir tomonlama sirtlar uchun $\vec{n}$ normal vektorning bunday ko'chishi dastlabki nuqtaga qaytilganda ( — $\vec{n}$ ) vektorga olib keladi.

Ma'lum tomoni tanlangan sirt $\sigma$ orientatsiyalangan deyiladi.

## 2. Ikkinchi tur sirt integrallari.

$\sigma^+$ — biror silliq sirt bo'lib, unda $\vec{n}$ = {cosa, cos$\beta$, cos-u} yo'nalish bilan xarakterlanuvchi musbat tomon tanlangan bo'lsin; R{x, u, z), Q (x, u, z), R(x, u, z) uzluksiz funksiyalar bo'lsin, u xolda mos ikkinchi tur sirt integrali quyidagicha ifodalanadi:

$$\iint_{\sigma^+} Pdydz + Qdzdx + Rdxdy = \iint_{\sigma} (Pcos\alpha + Qcos\beta + Rdxdy)d\sigma$$

Bu formula birinchi va ikkinchi tur sirt integrallarini o'zaro bog'laydi. Sirtning boshqa $\sigma^-$ tomoniga o'tilganda bu integral ishorasini qarama-qarshisiga o'zgartiradi. Agar $\sigma$ sirt z=z(x,y) tenglama bilan oshkor holda berilgai bo'lsa, u xolda $\vec{n}$ normalning yo'naltiruvchi kosinuslari quyidagi formulalar bo'yicha aniqlanadi:

cos$\alpha = \frac{1}{\pm|\vec{n}|} \cdot \frac{dz}{dx}; cos\beta = \frac{1}{\pm|\vec{n}|} \cdot \frac{dz}{dy}, cosy = \frac{1}{\pm|\vec{n}|}$ ,

bu yerda $|\vec{n}|$ = $\sqrt{1 + (\frac{dz}{dx})^2 + (\frac{dz}{dy})^2}$ va ishora tanlash sirt tomoni bilan muvofiqlashgan bo'lishi kerak.

Agar $\sigma$ sirt tenglamasi F(x, u, z) = 0 oshkormas xolda berilgan bo'lsa, bu sirt normali $\vec{n}$ ning yo'naltiruvchi kosinuslari quyidagi formulalar bo'yicha aniqlanadi:

cos$\alpha$=$\frac{1}{D} \cdot \frac{dF}{dx}$, cos$\beta = \frac{1}{D} \cdot \frac{dF}{dy}, cosy = \frac{1}{d} \cdot \frac{dF}{dz}$ bu yerda

D=$\pm\sqrt{(\frac{dF}{dx})^2 + (\frac{dF}{dy})^2 + (\frac{dF}{dz})^2}$ ва ildiz oldidagi ishorani

rani tanlash sirt tomoni bilan muvofiklashtirilishi kerak.Ikkinchi tur sirt integrali, shuningdek, koordinatalar bo'yicha sirt integrali deb ham ataladi.

Ikkinchi tur sirt integralini hisoblashni bevosita ikki o'lchovli integralni hisoblashga keltirish mumkin. Agar $\sigma$ sirt z — z(x, u) tenglamaga ega bo'lsa, u xolda ikkinchi tur sirt integrali quyidagi formula bo'yicha hisoblanadi:

$$\iint_{\sigma} R(x,y,z)\,dxdy = \pm \iint_{\sigma_{xy}} R(x,y,z)dxdy,$$

Bu yerda $\sigma_{xy}$сирт $\sigma$ ниг $O_{xy}$ текисликка проексияси.

$\pm$ *ishoralar sirtning ikkita turli tomonlariga mos keladi*: *bunda* « $+$» *ishora tanlangan tomonda cosy* $> 0$ *bo′lganida* « $-$» esa cosy$< 0$ *bo′lgandaolinadi.*

$\sigma$ *sirt* $y = y(x,z)$ yoki x=x(y,z) tenglamalar bilan berilgan hollarda qolgan integrallar ham xuddi yuqoridagidek hisoblanadi:

$$\iint_{\sigma} Q(x,y,z)dzdx = \pm \iint_{\sigma_{xz}} Q(x,y(x,z),z)dzdx,$$

bu yerda $\sigma_{xz}$ – sirt $\sigma$ ning $Oxz$ tekislikka proektsiyasi; <+> ishora tanlangan tomonda $\cos\beta > 0$ bo'lganda, <-> ishora esa $\cos\beta < 0$ bo'lganda olinadi.

$$\iint_{\sigma} P(x,y,z)dydz = \pm \iint_{\sigma_{yz}} P(x(y,z),y,z)dydz,$$

bu yerda $\sigma_{yz}$ – sirt $\sigma$ ning $Oyz$ tekislikka proektsiyasi; <+> ishora tanlangan tomonda $\cos\alpha > 0$ bo'lganda, <-> ishora esa $\cos\alpha < 0$ bo'lganda olinadi.

2-misol. Ushbu integralni hisoblang:

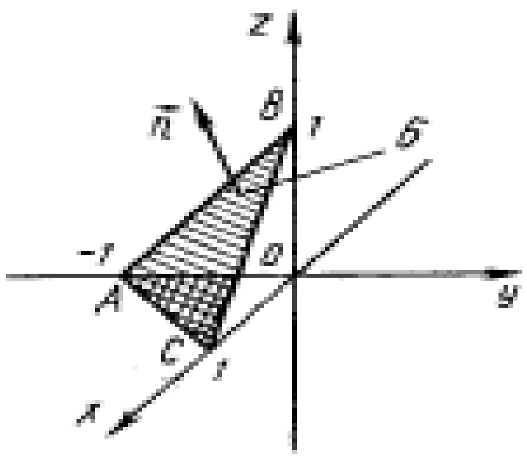
$$I = \iint_{\sigma} zdxdy + xdxdz + ydydz,$$

bu yerda $\sigma x\,y + z = 1$ tekislikning koordinata tekisliklari bilan kesishishdan hosil bo'lgan uchburchak; sirtning tanlangan tomonida normalg' $Oz$ o'qi bilan o'tkir burchak tashqil etadi.

Yechish. Shaklni chizamiz va integrallash tomonni $\underset{n}{\rightarrow}$ normali yordamida tanlashni ko'rsatamiz (61-shakl). $z = 1 - x + y$sirt tenglamasiga ega miz, $\frac{\partial z}{\partial x} = -1, \frac{\partial z}{\partial y} = 1, \cos\gamma > 0$,shuning uchun $\cos\alpha = -\frac{-1}{\sqrt{1+1+1}} = \frac{1}{\sqrt{3}}$;

$\cos\beta = -\frac{-1}{\sqrt{1+1+1}} = -\frac{1}{\sqrt{3}}$; $\cos\gamma = \frac{1}{\sqrt{3}}$.

Berilgan integralni hisoblash uchun quyidagi formulani hosil qilamiz:




61- шакл

$$I = \iint_{\sigma} zdxdy + xdxdz + ydydz = \iint_{\sigma} \left( y\frac{1}{\sqrt{3}} - x\frac{1}{\sqrt{3}} + y\frac{1}{\sqrt{3}} \right) d\sigma =$$

$$= \frac{1}{\sqrt{3}} \iint_{\sigma} \left((y-x)+z\right) d\sigma = \frac{1}{\sqrt{3}} \iint_{\sigma_{xy}} \left(y - x + (1 - x + y)\right) \sqrt{1+1+1}\ dxdy =$$

$$= \iint_{\sigma_{xy}} (2y - 2x + 1) dxdy,$$ бу ерда $\sigma_{xy}$ $\sigma$ сирт $(\sigma ABC)$нинг $Oxy$

Tekislikka proeksiyasi$(\Delta AOC)$. Ikki o'lchovli integralda chegaralarni qo'yib chiqamiz:

$$I = \iint_{\sigma_{xy}} (2y - 2x + 1) dxdy = \int_0^1 dx \int_{x-1}^{0} (2y - 2x + 1) dy =$$

$$= \frac{1}{4} \int_0^1 (2y - 2x + 1)^2 \Big|_{x-1}^{0} dx = \frac{1}{4} \int_0^1 \left((1-2x)^2 - 1\right) dx =$$

$$= \left( -\frac{1}{8}\frac{(1-2x)^3}{3} - \frac{x}{4} \right)\Big|_0^1 = \frac{1}{24} - \frac{1}{4} + \frac{1}{24} = -\frac{1}{6}.$$

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