

**MAKSIMUM BELGISI OSTIDA FUNKSIONAL PARAMETRNI O'Z  
ICHIGA OLGAN TENGLAMALAR SISTEMASI UCHUN  
BOSHLANG'ICH MASALA.**

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**Annotatsiya :** Ushbu maqolada turlarning chiziqli bo'lmagan integro-defferensial tenglamalar sistemasi ko'rib chiqiladi

**Kalit so'zlar** boshlang'ich shart, integro-defferensial tenglamalar tizimi, funksional parametr, Inter jarayonini, cheklangan yopiq ko'phadlar , segment.

**ИСХОДНАЯ ЗАДАЧА ДЛЯ СИСТЕМЫ УРАВНЕНИЙ, СОДЕРЖАЩАЯ  
ФУНКЦИОНАЛЬНЫЙ ПАРАМЕТР ПОД ЗНАКОМ МАКСИМУМА.**

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**Аннотация:** В статье рассматривается система нелинейных интегро-дифференциальных уравнений вида.

**Ключевые слова:** начальное условие, система интегро-дифференциальных уравнений, функциональный параметр, интерпроцесс, ограниченные замкнутые многочлены, отрезок.

**AN INITIAL PROBLEM FOR A SYSTEM OF INTEGRO-DIFFERENTIAL  
EQUATIONS CONTAINING A FUNCTIONAL PARAMETER UNDER  
THE SIGN OF THE MAXIMUM.**

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**Abstract:** The system of non-linear integro-differential equations of species is considered in this article.

**Keywords:** initial condition, system of integro-differential equations, functional parameter, Inter process, limited closed polynomials, segment.

Ushbu maqolada turlarning chiziqli bo'lmagan integro-defferensial tenglamalar tizimi ko'rib chiqiladi.

$$x'(t) = F \left( t, x(t), \int_0^t K(t, \theta) \max \{x(r) \mid r \in [\theta - h; \theta]\} d\theta, u(t) \right), t \geq 0 \quad (1)$$

Quydagicha boshlang'ich shart berilgan

$$x(t) = \varphi(t), \quad t \in E_0 \equiv [-\alpha, 0] \quad (2)$$

Bu yerda  $x \in X \subset R^n$  vektor holati,  $u \in U \subset R^m$  funksional parameter  $X, U$  cheklangan yopiq ko'phadlar,  $h = h(t, x(t), u(t))$  kechikish  $T$  vaqtiga bog'liq, kerakli  $x(t)$  funksiyadan va  $u(t)$  funksional parametrdan,  $K(t, \theta) \{0 \leq \theta \leq t \leq T\}$  shu oraliqda uzluksiz  $n \times n$  matrissali-funksiya,  $t - h(t, x, u) \geq -\alpha_0 = const$ , maximum. Quydagi ifodani soddalashtirish uchun biz quydagi belgilashni qabul qildik.

$$\rho = \int_0^t K(t, \theta) \max \{x(r) \mid r \in [\theta - h; \theta]\} d\theta, h_m = h(\theta, x_m(\theta), u(\theta)),$$

$$\rho_m = \int_0^t K(t, \theta) \max \{x_m(r) \mid r \in [\theta - h_m; \theta]\} d\theta, \int_0^t \|K(t, \theta)\| d\theta \leq \alpha < \infty$$

Keling, nima sodir bo'lishini isbotlaylik

**Teorema. Quyidagilar o'rinli bo'lsin:**

$$1 \quad F(t, x, \rho) \in C([0; T] \times X \times R^n \times U) \cap B_{nd}(M) \cap Lip(L_{1, X, \rho}); \quad (3)$$

$$2 \quad t - h(t, x, u) \geq -\alpha_0 = const \quad (4)$$

$$\text{Va } h(t, x, u) \in C([0; T] \times X \times U) \cap Lip(L_{2, X}) \quad (5)$$

$$3. \varphi(t) \in Lip(L_3) \quad (6)$$

Bu yerda  $X = \{x \in R^n \mid \|x - \varphi(0)\| \leq r\}, r = \max \{\|\varphi(t) - \varphi(0)\| \mid t \in E_0\}$ .

Keyin  $\left[0; t^*\right], t \leq T$  segmentda (2) boshlang'ich shart bilan  $x(t) \in X$  funksiyaning yagona yechimi mavjud.

**Isbot.** Biz ketma-ket yondashuvlar yordamida (1) tenglamani (2) ga aylantiramiz.

$$\left\{ \begin{array}{l} x_0(t) = \varphi(t), t \in E_0, x_0(t) = \varphi(t), t \geq 0 \\ x_{m+1}(t) = \varphi(t), t \in E_0, m = 0, 1, 2, \dots \\ x_{m+1}(t) = \varphi(t) + \int_0^t F(\theta, x_m(\theta), \rho_m(\theta), u(\theta)) d\theta, t \geq 0 \end{array} \right. \quad (7)$$

Barcha yondashuvlar  $X$   $t \in \left[0; t^*\right]$  da qolishiga ishonch hosil qilish qiyin emas.

Inter jarayonining farqini baholaymiz (7) da .  $x_1(t) - x_0(t)$  farqi uchun (3) hisobga olinsa , biz quydagicha baholaymiz.

$$\|x_1(t) - x_0(t)\| \leq \bar{M}t, t \in \left[0; t^*\right] \quad (8)$$

Bundan tashqari farq uchun  $x_2(t) - x_1(t)$  bizda bor

$$\|x_1(t) - x_0(t)\| \leq L_1 \int_0^t (\|x_1(\theta) - x_0(\theta)\| + \|\rho_1(\theta) - \rho_0(\theta)\|) d\theta. \quad (9)$$

Baholash uchun  $\rho_1(t) - \rho_0(t)$  biz bu farqni quydagicha yozamiz

$$\begin{aligned} \rho_1(t) - \rho_0(t) &= \int_0^t K(t, \theta) (\max \{x_1(r) \mid r \in [\theta - h_1; \theta]\} - \\ &- \max \{x_0(r) \mid r \in [\theta - h_0; \theta]\}) d\theta = \int_0^t K(t, \theta) (\max \{x_1(r) \mid r \in [\theta - h_1; \theta]\} - \\ &- \max \{x_0(r) \mid r \in [\theta - h_1; \theta]\}) - (\max \{x_0(r) \mid r \in [\theta - h_1; \theta]\} - \\ &- \max \{x_0(r) \mid r \in [\theta - h_0; \theta]\}) d\theta. \end{aligned}$$

(10)

(8) tenglikning birinchi farqi uchun (10) hisobga olinsa, quydagicha olish mumkin

$$\begin{aligned} &\left\| \max \{x_1(r) \mid r \in [t - h_1; t]\} - \max \{x_0(r) \mid r \in [t - h_1; t]\} \right\| \leq \\ &\leq \left\| \max \{(x_1(\tau) - x_0(\tau)) \mid \tau \in [t - h_1; t]\} \right\| \leq Mt \\ &\max \{x_1(\tau) \mid \tau \in [t - h_1; t]\} = \max \{(x_1(\tau) - x_0(\tau) + x_0(\tau)) \mid \tau \in [t - h_0; t]\} \leq \\ &\leq \max \{(x_1(r) - x_0(r)) \mid r \in [t - h_1; t]\} + \max \{x_0(r) \mid r \in [t - h_1; t]\} \\ &\max \{x_1(r) \mid r \in [t - h_1; t]\} - \max \{x_0(r) \mid r \in [t - h_1; t]\} \leq \\ &\leq \max \{(x_1(r) - x_0(r)) \mid r \in [t - h_1; t]\} \leq \\ &\leq \max \{(x_1(r) - x_0(r)) \mid r \in [t - h_1; t]\} \\ &\max \{x_0(r) \mid r \in [t - h_1; t]\} - \max \{x_1(r) \mid r \in [t - h_1; t]\} \leq \\ &\leq \max \{(x_0(r) - x_1(r)) \mid r \in [t - h_1; t]\}, \quad t \geq 0 \end{aligned}$$

Xuddi shunday, biz quyidagi natijani olamiz

$$\begin{aligned} &\max \{x_0(r) \mid r \in [t - h_1; t]\} - \max \{x_1(r) \mid r \in [t - h_1; t]\} \leq \\ &\leq \max \{(x_0(r) - x_1(r)) \mid r \in [t - h_1; t]\}, \quad t \geq 0 \end{aligned}$$

Ushbu tengsizlikni chap va o'ng qismidan maksimum  $t$  ni olib, biz isbotlashni, istalgan tengsizlikni qo'lga kiritamiz. (4)-(6) ni hisobga olgan holda, o'ng tengsizlikdagi ikkinchi farq uchun quyidagi farqni olamiz.

$$\begin{aligned} & \left\| \max \{x_0(r) \mid r \in [t-h_1; t]\} - \max \{x_0(r) \mid r \in [t-h_0; t]\} \right\| \leq \\ & \leq L_3 \|h(t, x_1(t), u(t)) - h(t, x_0(t), u(t))\| \leq \\ & \bar{M}L_2 \|x_1(t) - x_0(t)\| \leq \bar{M}^2 L_2 t, \bar{M} = \max \{M, L_3\} \\ & \|\rho_1(t) - \rho_0(t)\| \leq \bar{M}\alpha(1 + ML_2)t \end{aligned}$$

Shuning uchun

$$\|\rho_1(t) - \rho_0(t)\| \leq \bar{M}\alpha(1 + ML_2)t$$

Keyin (9) quyidagicha yoziladi:

$$\begin{aligned} \|x_2(t) - x_1(t)\| & \leq L_1 \int_0^t \bar{M} [1 + \alpha(1 + ML_2)] \theta d\theta = L_1 \bar{M} [1 + \alpha(1 + ML_2)] \frac{t^2}{2!} \\ \|x_3(t) - x_2(t)\| & \leq L_1 \int_0^t (\|x_2(\theta) - x_1(\theta)\| + \rho_2(\theta) - \rho_1(\theta)) d\theta \leq \\ & \leq L_1 \left[ 1 + \alpha \left( 1 + \|x_1'(\theta)\| \right) L_2 \right] \int_0^t L_1 \bar{M} [1 + \alpha(1 + ML_2)] \frac{\theta}{2!} d\theta = \\ & = L_1^2 \bar{M} [1 + \alpha(1 + ML_2)]^2 \frac{t^3}{3!} \end{aligned}$$

$x_3(t) - x_2(t)$  farqni quyidagicha olamiz

$$\begin{aligned} \|x_3(t) - x_2(t)\| & \leq L_1 \int_0^t (\|x_2(\theta) - x_1(\theta)\| + \rho_2(\theta) - \rho_1(\theta)) d\theta \leq \\ & \leq L_1 \int_0^t (\|x_2(\theta) - x_1(\theta)\| + \int_0^\theta \|K(\theta, \xi)\| \\ & [\|\max \{x_2(r) \mid r \in [\xi - h_2, \xi]\} - \max \{x_1(r) \mid r \in [\xi - h_2, \xi]\}\| + \\ & + \|\max \{x_1(r) \mid r \in [\xi - h_2, \xi]\} - \max \{x_1(r) \mid r \in [\xi - h_1, \xi]\}\|] d\xi) d\theta \leq \\ & = L_1^2 \bar{M} [1 + \alpha(1 + ML_2)]^2 \frac{t^3}{3!} \end{aligned}$$

Demak, ushbu baholashlardan  $m \rightarrow \infty$  da  $\|z(t) - x_m(t)\| \rightarrow 0$  manashu  $[0; t^*]$  segmentdagi  $t$  da teng ravishda ko'rinadi. Shundan kelib chiqadiki,  $\{x_m(t)\}$  ketma-ketligini talab qiladigan (1) tenglamaning yechimi  $[0; t^*]$  segmentida yagona. **Teorema** isbotlandi.

#### Adabiyotlar

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