DIFFERENSIAL TENGLAMA YECHIMINIMG MAVJUDLIGI VA YAGONALIGI. MAXSUS YECHIM TUSHUNCHASI

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Annotatsiya: Differensial tenglamalar nazariyasi va uning amaliy tatbiqlari uchun Koshi masalasi yechimning mavjudligi va yagonali katta ahamiyatga ega. Ushbu ishda maxsus yechim tushunchasi va unga oid ba’zi misollar keltirilgan.

Kalit so‘zlar: differensial tenglama, funksiya, umumiy integral, Koshi masalasi, maxsus nuqta, maxsus yechim.

EXISTENCE AND UNIQUENESS OF THE SOLUTION OF THE DIFFERENTIAL EQUATION. SPECIAL SOLUTION CONCEPT

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Annotation. For the theory of differential equations and its practical applications, the existence and uniqueness of a solution to the Cauchy problem is of great importance. This paper presents the concept of a special solution and some examples of it.

Keywords. differential equation, function, general integral, Cauchy problem, singular point, particular solution.

Differensial tenglama umumiy va xususiy yechimi tushunchasida aytiladiki, sohaning har bir berilgan nuqtasidan Koshi masalasining yagona yechimi o‘tadi. Shunday qilib umumiy yoki xususiy yechimning har bir nuqtasida Koshi masalasi yagona yechimga ega bo‘ladigan nuqta bo‘ladi. Ayniqsa, differensial tenglamani taqribiy yechish usullaridan foydalanish uchun berilgan boshlang‘ich shartni qanoatlantiruvchi yechimning mavjudligi va yagonaligiga ishonch hosil qilish muhim hisoblanadi.

Hosilaga nisbatan yechilgan

$$\frac{dy}{dx}=f(x,y) \tag{1}$$

differensial tenglamani qaraymiz.

Teorema (mavjudlik va yagonalik). Faraz qilaylik, $f(x,y)$ funksiya tekislikdagi $D=\{(x,y):|x-x_0|\le a,|y-y_0|\le b\}$ to‘plamda uzluksiz va

$$|f(x,y_1)-f(x,y_2)|\le N|y_1-y_2| \qquad (2)$$

Lipshits shartini qanoatlantirsin, bu yerda $N=const$. U vaqtda (2.11) tenglamaning $[x_0-H,x_0+H]$ oraliqda anqlangan va$y(x_0)=y_0$ boshlang‘ich shartni qanoatlantiruvchi $y=y(x)$ yechimi mavjud va yagonadir, bu yerda

$$H<min\left\{a,\frac{b}{M},\frac{1}{N}\right\},\ M=\max_{(x,y)\in D} f(x,y).$$

Eslatma. 1. Agar $f(x,y)$ funksiya D to‘plamda faqat uzluksiz bo‘lib, Lipshits sharti bajarilmasa, u vaqtda (1) tenglamaning $y(x_0)=y_0$ shartni qanoatlantiruvchi yechimi mavjud bo‘lsada, bunday yechim yagona bo‘lmasligi mumkin.

1. Keltirilgan teoremada Lipshits shaartini $f'_y(x,y)$ xususiy hosilaning D to‘plamda usluksizligi yoki modul bo‘yicha chegaralanganligi bilan almashtirish mumkin. Haqiqatan ham, chekli orttirmalar haqidagi teoremaga ko‘ra ixtiyoriy $(x,y_1)\in D$,$(x,y_2)\in D$ uchun

$$|f(x,y_1)-f(x,y_2)|=|f'_y(x,\xi)||y_1-y_2|,\xi\in[y_1,y_2] \qquad (3)$$

bo‘ladi. $(x,\xi)\in D$uchun $|f'_y(x,\xi)|\le N$ bo‘lgani uchun (3) dan (2.) Lipshits sharti kelib chiqadi.

Ta’rif 1. Agar (1) tenglamaning (x_0,y_0) nuqta atrofida $y(x_0)=y_0$ shartni qanoatlantiruvchi yechimi mavjud emas, yoki yagona bo‘lmasa, bunday nuqta differensial tenglama uchun ***maxsus nuqta*** deb ataladi.

Ta’rif 2. Agar (1) tenglamaning $y=y(x)$ yechimi grafigi faqat maxsus nuqtalardan tashkil topgan bo‘lsa, u vaqta bu yechimga ***maxsus yechim*** deb aytiladi.

Maxsus nuqta va maxsus yechim mavjudlik va yagonalik teoremasi shartlari buziladigan nuqtalar orasida bo‘lsadi.

Mavjudlik va yagonalik teoremasining birinchi sharti $f(x,y)$ funksiyaning uzilish nuqtalarida buziladi. Agar differsial tenglamga keltirilgan masalada x va y

o‘zgaruvchilar teng huquqli bo‘lsalar, mavjudlik va yagonalik teoremasi birinchi sharti $f(x,y)$ va $\frac{1}{f(x,y)}$ funksiyalar bir vaqtda uzilishga ega bo‘ladigan nuqtalarda buziladi. Agar (2.5) tenglamada $f(x,y)=\frac{M(x,y)}{N(x,y)}$, $M(x,y), N(x,y)$– uzluksiz funksiyalar bo‘lsa, faqat $M(x_0,y_0)=N(x_0,y_0)=0$ va $\lim_{\substack{x\to x_0\\ y\to y_0}}\frac{M(x,y)}{N(x,y)}$,$\lim_{\substack{x\to x_0\\ y\to y_0}}\frac{N(x,y)}{M(x,y)}$ chekli limitlar mavjud bo‘lmagan holda (x_0,y_0) maxsus nuqta bo‘ladi.

Mavjudlik va yagonalik teoremasining ikkinchi muhum sharti bo‘lgan Lipshits sharti yoki $f'_y(x,y)$ xususiy hosilaning chegaralanganlik sharti $\frac{1}{f'_y(x,y)}=0$ tenglik bajariladigan nuqtalarda buziladi. $\frac{1}{f'_y(x,y)}=0$ tenglamadan odatda qandaydir chiziq yoki chiziq shoxchalari aniqlanadi. Agar shu chiziq shoxchalaridan birortasi (1) tenglama yechimi bo‘lib, unung nuqtalarida yechim mavjudligi buzilsa, u vaqtda bu shoxcha tenglamaning maxsus yechimini ifodalaydi.

Ushbu

$$y'=\sqrt{y}$$

Differensial tenglamni qaraylik. Bu tenglama $D=\{(x,y)\in R^2: -\infty<x<+\infty, 0<y<+\infty\}$ sohada ushbu

$$y(x)=\frac{1}{4}(x+C)^2, x\geq -C$$

ko’rinishdagi umumiy yechimga ega. Berilgan differensial tenglama uchun $y(x)\equiv 0$ maxsus yechim bo’ladi. Haqiqatan ham, ixtiyoriy $M(x_0,0)\in R$ nuqtadan berilgan differensial tenglamaning kamida ikkita

$$y_1(x)\equiv 0, y(x)=\begin{cases} 0, x\leq x_0 \\ (x-x_0)^2, x>x_0 \end{cases}$$

yechimi o’tadi.

Quyidagi

$$\sqrt{4+y^2}\,dx-(x-1)\,ydy=0$$

differensial tenglamani qaraylik, tenglamani ko’rinishini o’zgartiramiz, buning uchun tenglama ikkala tomonini $\sqrt{4+y^2}\cdot(x-1)$ ga bo’lsak,

$$\frac{dx}{x-1}-\frac{ydy}{\sqrt{4+y^2}}=0, x\neq 1,$$

$$\int\frac{dx}{x-1}-\int\frac{ydy}{\sqrt{4+y^2}}=C,$$

yoki

$$\ln|x-1|-\sqrt{4+y^2}=C$$

tenglamani umumiy integralga ega bo’lamiz. Bundan tashqari $x=1$ ham berilgan tenglamaning yechimi bo’ladi chunki $0=0$ bajariladi.

Bundan ko‘rinadiki, maxsus yechim, differensial tenglamaning xususiy yechimi bo‘la olmaydi va u umumiy yechim formulasi tarkibiga ham kirmaydi.

Foydalanilgan adabiyotlar.

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