O'RTA GEOMETRIK, O'RTA GARMONIK VA GYOL'DER TENGSIZLIKLAR ORASIDAGI MUNOSABATLAR

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Annotatsiya: Mazkur ishda shu maqsadga qaratilgan bo'lib, asosan geometrik tengsizliklar yordamida yechish mumkin bo'lgan geometrik masalalarga qaratilgan bo'lib, ularni batafsil yechish usullari keltirilgan va ularni qo'llab masalalar yechilgan. Geometriya bo'yicha ba'zi masalalarni,masalan, eng katta yoki eng kichik miqdorlarni topishga oid masalalarni yechish uchun ko'p jihatdan ularni yechish tajribasiga va yechish usullarini o'zlashtirganlik darajasiga bog'liq. Shuning uchun berilgan qarab qaysi tengsizlikni qo'llash va uni isbotlash muhimdir.

Kalit so'zlar: Geometriya, tengsizlik, tranzitivlik, o'rtacha qiymat, o'rta arifmetik, o'rta geometrik ,Koshi tengsizligi, o'rta garmonik, o'rta kvadratik qiymatlar, Gyol'der tengsizligi, Minkovskiy tengsizligi.

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Annotation: This work is aimed at this goal, mainly focused on geometric problems that can be solved using geometric inequalities, detailed methods for solving them are presented and problems are solved using them. Solving some problems in geometry, for example, problems related to finding the largest or smallest quantities, largely depends on the experience of solving them and the level of mastering the solution methods. Therefore, it is important to use which inequality to use and prove it depending on the given.

Keywords: Geometry, inequality, transitivity, average value, arithmetic mean, geometric mean, Cauchy inequality, harmonic mean, squared mean, Holder inequality, Minkowski inequality.

O'rta geometrik va o'rta garmonik qiymatlar orasidagi tengsizlik.

1-Teorema. $G(a) \geq H(a)$ ekanligini, jumladan, $H(a) = G(a)$ tenglik faqat va faqat $a_1 = a_2 = \dots = a_n$ shart bajarilsa to'g'riligini isbotlang.

Isboti. Koshi tengsizligidan foydalanib foydalanib $(H(a))^{-1} = \dfrac{a_1^{-1} + a_2^{-1} + \dots + a_n^{-1}}{n} \geq \sqrt[n]{a_1^{-1} a_2^{-1} \dots a_n^{-1}} = (G(a))^{-1}$ tenglikga ega bo'lamiz. Jumladan, $H(a) = G(a)$ tenglik faqat $a_1 = a_2 = \dots = a_n$ da bajariladi.

1-misol. Agar $a, b, c > 0$ bo'lsa, $\dfrac{3}{1/a + 1/b + 1/c} \leq \dfrac{a+b+c}{3}$ tengsizlikni isbotlang.

Yechilishi: $9 \le (a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$,

$$\begin{cases} a+b+c \ge 3\sqrt[3]{abc}, \\ \frac{1}{a}+\frac{1}{b}+\frac{1}{c} \ge 3\cdot\frac{1}{\sqrt[3]{abc}}. \end{cases} \Rightarrow (a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \ge \frac{9\sqrt[3]{abc}}{\sqrt[3]{abc}} = 9.$$

2-misol. Agar $a, b, c > 0$, $ab^2c^3 = 1$ bo'lsa, $\frac{1}{a}+\frac{1}{b}+\frac{1}{c} \ge 6$ ni isbotlang.

Yechilishi: $\frac{1}{a}+\frac{2}{b}+\frac{3}{c} = \frac{1}{a}+\frac{1}{b}+\frac{1}{b}+\frac{1}{c}+\frac{1}{c}+\frac{1}{c} \ge 6\frac{1}{\sqrt[6]{ab^2c^3}} = 6$.

O'rta arifmetik va o'rta kvadratik qiymatlar orasidagi tengsizlik.

2-Teorema. $K(a) \ge A(a)$ tengsizlik o'rinli ekanligini, jumladan, $K(a) - A(a)$ tenglik faqat $a_1 = a_2 = \dots = a_n$ holdagina o'rinli bo'lishini isbotlang.

Isboti: Koshi tengsizligidan foydalanib (1-masalaga qarang) foydalanib $2a_ia_j \le a_i^2 + a_j^2$, $1 \le i < j \le n$ tengsizlikni hosil qilamiz. Demak,

$$(a_1 + a_2 + \dots + a_n)^2 = a_1^2 + a_2^2 + \dots + a_n^2 + 2\sum_{1 \le i < j \le n} a_ia_j \le$$

$$\le a_1^2 + a_2^2 + \dots + a_n^2 + \sum_{1 \le i < j \le n} (a_i^2 + a_j^2) - n(a_1^2 + a_2^2 + \dots + a_n^2).$$

Eslatib o'tamiz, $K(a) - A(a)$ tenglik faqat $a_1 - a_2 - \dots - a_n$ o'rinli bo'ladi.

Gyol'der tengsizligi.

Teorema. $\frac{1}{p}+\frac{1}{q} = 1$ shartni qanoatlantiruvchi barcha musbat p, q sonlar va a_j, b_j, $j = 1, \dots, n$ sonlar uchun

$$\left|\sum_{i=1}^{n} a_ib_i\right| \le \left(\sum_{i=1}^{n} |a_i|^p\right)^{\frac{1}{p}} \left(\sum_{i=1}^{n} |b_i|^q\right)^{\frac{1}{q}} \qquad (4)$$

tengsizlik har doim to'g'ri.

Isboti. $\sum_{i=1}^{n} |a_i|^p \ne 0, \sum_{i=1}^{n} |b_i|^q \ne 0$ deb faraz qilamiz (aks holda (4) tengsizlik bajarilishi ravshan). Yung tengsizligini qo'llab

$$\left|\sum_{i=1}^{n} \frac{a_i}{\left(\sum_{k=1}^{n}|a_k|^p\right)^{\frac{1}{p}}} \frac{b_i}{\left(\sum_{k=1}^{n}|b_k|^q\right)^{\frac{1}{q}}}\right| \le \left|\sum_{i=1}^{n} \frac{|a_i|}{\left(\sum_{k=1}^{n}|a_k|^p\right)^{\frac{1}{p}}} \frac{|b_i|}{\left(\sum_{k=1}^{n}|b_k|^q\right)^{\frac{1}{q}}}\right| \le$$

$$\le \left|\sum_{i=1}^{n} \frac{|a_i|^p}{p\sum_{k=1}^{n}|a_k|^p} \frac{|b_i|^q}{q\sum_{k=1}^{n}|b_k|^q}\right| = \frac{1}{p} + \frac{1}{q} = 1$$

ga ega bo'lamiz. Bu yerdan (4) tengsizlik kelib chiqadi.

Izoh. Gyol'der tengsizligining

$p = q = 2$ dagi $\left|\sum_{i=1}^{n} a_i b_i\right| \le \sqrt{\sum_{i=1}^{n} a_i^2}\sqrt{\sum_{i=1}^{n} b_i^2}$ *Koshi-Bunyakovskiy-Shvarts* tengsizligi deb ataluvchi bir muhim hususiy holini aytib o'tamiz.

1-misol (Minkovskiy tengsizligi). Ixtiyoriy musbat a_j, b_j $(j = 1,...,n)$ sonlar va natural p son uchun

$$\left(\sum (a_k + b_k)^p\right)^{1/p} \le \left(\sum a_k^p\right)^{1/p} \left(\sum b_k^p\right)^{1/p} \qquad (5)$$

tengsizlikni isbotlang.

Yechilishi. $(a_k + b_k)^p = a_k (a_k + b_k)^{p-1} + b_k (a_k + b_k)^{p-1}$ $(k=1, 2, ..., n)$ tengsizlikni qo'shib, $\sum (a_k + b_k)^p = \sum a_k (a_k + b_k)^{p-1} + \sum b_k (a_k + b_k)^{p-1}$ ni olamiz. (4) tengsizlikka ko'ra

$$\sum a_k (a_k + b_k)^{p-1} \le \left(\sum a_k^p\right)^{1/p} \left(\sum (a_k + b_k)^{q(p-1)}\right)^{1/q},$$

$$\sum b_k (a_k + b_k)^{p-1} \le \left(\sum b_k^p\right)^{1/p} \left(\sum (a_k + b_k)^{q(p-1)}\right)^{1/q}$$

larga ega bo'lamiz, bu yerdan $q(p-1) - p$ tenglik yordamida (6) tengsizlik kelib chiqadi.

3.-masala. $x^2 + y^2 + z^2 \ge xy + xz + yz$ tengsizlikni isbotlang x, y, z musbat sonlar.

Yechilishi. Ma'lum $x^2 + y^2 \ge 2xy$, $x^2 + z^2 \ge 2xz$, $y^2 + z^2 \ge 2yz$ tengsizliklarni qo'shib, ushbu $(x^2 + y^2) + (x^2 + z^2) + (y^2 + z^2) \ge 2(x^2 + y^2 + z^2) \ge 2(xy + xz + yz)$ tengsizlikni olamiz.

4-masala. $x^4 + y^4 + z^4 \ge xyz(x + y + z)$ tengsizlikni isbotlang, bu yerda x, y, z - musbat sonlar.

Yechilishi. 1-masalaga ko'ra:
$x^4 + y^4 + z^4 = (x^2)^2 + (y^2)^2 + (z^2)^2 \ge x^2 y^2 + y^2 z^2 + x^2 z^2$ ga egamiz.

Bu yerdan esa $x^2 y^2 + y^2 z^2 + x^2 z^2 \ge xyyz + yzzx + zxxy = xyz(x + y + z)$ ni olamiz.

5.-masala. $x^4 + y^4 + z^4 + u^4 \ge 4xyzu$ tengsizlikni isbotlang, bu yerda x, y, z, u - musbat sonlar.

Yechilishi. $x^4+y^4\ge 2x^2y^2$, $z^4+u^4\ge 2z^2u^2$ ga egamiz. Demak, $x^4+y^4+z^4+u^4\ge 2x^2y^2+2z^2u^2$. Bundan tashqari $x^2y^2+z^2u^2\ge 2xyzu$. Demak, $x^4+y^4+z^4+u^4\ge 4xyzu$.

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