A CONTRIBUTION TO OPTIMAL TAXATION THEORY

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Abstract

Currently in our globalized world and modernized societies, economic development is becoming one of the main goals of economic policymakers in order to provide nations with highest possible level of prosperity and improved living standards. Public economics as being one of the important parts of the economy, in recent years, has, to some extent, an impact on economic development. Specifically, core part of the public economics, taxation, have been influencing growth path of the economy since the first initiation of it. With respect to this, current paper mainly tries to contribute to the optimal tax theory literature by trying to establish a mathematical model that can calculate optimal level of different types of tax rates, such as personal income tax and consumption tax, which eventually leads to steady state level of economic growth in the long run. As it already obvious, by optimal it is meant that the certain level of tax rate that may push economic growth to its golden steady state level in the long run[1]. An undisputable fact that needs to be included here is that the mathematical model constructed mainly based on golden rule of saving of basic Solow model of economic growth along with basic consumption/saving theory of macroeconomics. Specifically, mathematical model tries to develop basic Solow model of economic growth by adjusting it for possible effects of public economics and taxation. Besides, noteworthy point to emphasize here is that taxation can be a very useful tool that can increase aggregate saving level in the economy towards its golden rate, especially in developing countries, by properly controlling and resetting different types of tax rates based on the households' and government's marginal propensity to save.

Key words: economic growth, GDP per capita, taxation, personal income tax, consumption tax, optimal tax theory, mathematical model.

JEL classifications: C02, C23, F63, H21, O11.

1. Introduction and motivation

[1] For detailed information regarding golden steady-state level of economic growth see Robert M. Solow, 'A Contribution to the Theory of Economic Growth', *Quarterly Journal of Economics*, 70, 1956. and Sorensen P. and Jacobson H., 'Introducing Advanced Macroeconomics: Growth and Business Cycles', 2010, pp. 78-79.

While economics, being a social science[2], is all about finding possible ways of dealing with the fundamental problem of conciliating unlimited desires of individuals with scarce resources, numerous approaches have been revealing specific objectives of the subject, such as economic growth, stability of balance of payments, redistribution of income, price stability, full employment and many others. Alternatively, it is also possible to state that economics is an art of distributing limited resources in an optimal way that maximizes utility of every single participant of an economy in order to reach the highest level of prosperity[3] in the society, while all individuals are concentrated on behaving rationally with an eye to get greater satisfaction from life or in economic language trying to maximize their utility. From the perspective of microeconomics, generally, an individual maximizes his utility by consuming preferable variety of goods and services in an optimal quantity subject to the prices of commodities and his/her income, which is determined by individual's labor supply who tries to optimally balance his/her working and non-working (leisure) time. However, according to macroeconomic point of view, where all indicators measured in aggregate level, welfare of the society could be increased by optimally allocating aggregate income into consumption and saving, which eventually leads to the highest level of prosperity through its effect on capital accumulation.

While optimal tax theory along with various growth models have long been embarrassing economic policymakers and fascinating economic theorists, this paper mainly concentrates on exploring the role of taxation in the process of reaching economic growth. Specifically, it tries to initiate a mathematical model that is addressed to calculate optimal rates of different types of taxes in order to achieve long run economic growth based on basic Solow model of economic growth[4]. Government by possessing controlling and monitoring role in the economy has, to some extent, an engagement to provide population with public goods and services[5], for example education and healthcare, legal system and defense, as well as infrastructure like roads. In order to fund these expenses government needs to obtain some financial assets. Despite other forms of revenues, such as income from certain public goods and services, or from sales of government assets and bonds, and maybe even from the opportunity of borrowing from

[2] Social science studies behavior of society and of individuals
[3] Prosperity is high level of life standards with successful social status and profuse wealth, such as health and happiness.
[4] Robert M. Solow, 'A Contribution to the Theory of Economic Growth', *Quarterly Journal of Economics*, 70, 1956.
[5] Public goods and services are the goods and services that are not supplied by private sector of market due to the fact of inefficient production process and lower rate of profitability, thus being produced by public sector.

international organizations, like World Bank or International Monetary Fund, levying taxes remains as the main source of government budget. Hence, taxation is required with an eye to finance public expenditure.

Current research mainly concentrates on first function of the government, economic stabilization, by trying to reveal optimal level of various types of tax rates, such as personal income tax and consumption tax that can eventually lead to the long run steady state level of economic growth. In addition, the model also tries to balance the rates of different types of taxes by calculating their optimal levels as it mentioned above. However, standard theory of optimal taxation affirms that a tax system that considered as optimal one should be constructed to maximize social welfare subject to the utility function and consumption decisions of households in the economy (Mankiw, Weinzierd, Yagan, 2009). As it mentioned above, therefore, the model of the research is based on the basic Solow model of economic growth and believed to calculate optimal level of different tax rates that can lead to optimal level of long run economic growth. It is essential to emphasize that great scientist and economist Robert Merton Solow in his work in 1956 analyzed a possible theoretical way of reaching optimal level economic growth that maximizes share of consumption in the income in the long run. Based on his research it is possible to mention that aggregate consumption of a nation reaches its maximum level, when national saving rate in the economy equals to the share of aggregate capital stock in the production process[6]. Tax system, thanks to its flexibility feature, can influence consumption and saving decisions of households. To be more specific, it is an undisputable fact that almost all of the modern tax systems consist of mixture of various types of taxes, by allowing different forms of taxes to impact the economy in different ways, in order to increase influential power of tax system to the economy. Hence, the role of taxation in the process of reaching maximum level of aggregate consumption is going to be the subject of this research. Precisely, the question what should the rates of different taxes be in order to achieve golden rate of economic growth[7] in the long run, will be answered with the help of the

[6] See golden rule of basic Solow model of economic growth available from: Sorensen P. and Jacobson H., 'Introducing Advanced Macroeconomics: Growth and Business Cycles', 2010, pp. 78-79.

[7] Golden rate of economic growth is a certain level of economic growth in which the gap between the aggregate income and national saving, level of aggregate consumption, becomes maximum in the long run as a result of equalizing saving rate to the share of capital stock in the production process.

mathematical model. Besides, the model also tries to balance the rates of various types of taxes in the economy for stabilization purposes.

2. Literature review

Since the latter half of the previous century optimal taxation has become one of the most analyzed and examined topics in the literature on public economics. To be more species, public economics is mainly concentrated on the study of economic policy, with specific focus on taxation. As a matter of fact, public economics have been existing for almost more than two centuries, as Ricardo (1817) initiated some of his ideas regarding possible influences of public debt on general economic performance. Later on, Cournot (1838) conducted a research regarding origins of taxation in the imperfectly competitive markets, whereas impact of taxation on multiproduct companies was main interest of Edgeworth (1925). Besides, great contribution to this field of study made by Pareto (1909) by designing basics to efficient social decision making process. It is worthy stating that there needs to be developed an appropriate theory before implementing an adequate policy. However, according to Myles (1995) public economic is a challenging subject, since there is a need on accomplishing considerable work because major part of this field of subject still in the level of infancy. One of this type of subject areas is designing optimal taxation that makes economic performance in its highest level. By economic performance, many different concepts can be understood, such as sustained economic growth, equal distribution of income among individuals, decreased poverty rate, efficient production process and many others. In fact, current paper tries to design an optimal taxation rate taking into consideration sustained economic growth and increasing level of prosperity. Before analysis of effect of taxation on economic development, it seems to be efficient to review some interesting studies regarding optimal taxation theory. We designed our review of literature first separately discussing optimal taxation theory in personal income tax and consumption tax level, which are main interest of this research, and then we introduce combined optimal taxation theory. Once, Myles emphasized that “the motivation for the study of public economics follows naturally from the observation that unregulated economic activity does not lead to a socially optimal outcome”.

2.1. Consumption tax

According to the literature on the topic of optimal commodity taxation, or consumption tax, certain level of government revenue needs to be collected solely from commodity taxation. In addition, the problem most of the scholars tried to solve is that how should the structure of taxation needs to be in order to minimize the cost of collecting this required level of revenue to the society. There are several studies done with an eye to come up with possible design of optimal structure of commodity taxation, which solves above stated problem. To begin with, Ramsey (1927) introduced the first and the oldest formal results. Specifically, according to him equity should not be considered in the process of setting the tax rate, which eventually results in efficient tax system. As a matter of fact, Ramsey rule was the core of this subject area until initiation of inverse elasticities rule after approximately four decades. Possible limitation of the study of Ramsey was that he introduced his rule based on single-household economy, which cannot meet all the real life economic conditions. Furthermore, Samuelson (1986) also supported the idea of Ramsey by re-revealing the results for the case of US Treasury. An important study to mention here is the research of Diamond and Mirrlees (1971), which upgraded commodity taxation to its modern form. More precisely, they mostly employed emerging duality methods in their analysis. Besides, it is also possible to emphasize that Diamond and Mirrlees (1971), differently from Ramsey (1927), revealed their results based on both single-household and many-household economy. To be more specific, their analysis results in general equilibrium theory and they also proved Production Efficiency Lemma. Since the initiation of inverse elasticities rule by Diamond and Mirrlees, almost all of the studies have been taking the form of empirical and more practical implementation of inverse elasticities rule.[8]

However, there is no clear information regarding exact level of government revenue that needs to be collected from commodity taxation, despite the possibility of designing optimal structure of taxation system that results in socially optimal outcome. To be more specific, almost all of the theoretical papers regarding optimal commodity taxation are in microeconomic level. However, there is an important role of dynamics of macroeconomic factors as well. More clearly, what share of government revenue needs to be collected with the help of consumption tax and what are the possible determinants of this. Alternatively, what kind of forces or factors

[8] For more detailed analysis and explanation of the papers, see Myles G., 'Public economics', 1995.

determine how many percentage of government revenue is supposed to be collected from the consumption of households? What are the main criteria for designing such a study? In fact, there is no such information in the literature of optimal commodity taxation. Hence, because of aforementioned uncertainties of the literature current paper will introduce possible method of calculating optimal level of consumption tax in terms of achieving sustained economic growth based on Solow model of economic growth.

2.2. Personal income tax

Since the establishment of income taxation, it has been becoming a major source of government tax revenue almost in all emerging economies and in some developing ones as well. Hence, optimal income taxation is one of the commonly analyzed concepts in the public economics literature. On the one hand, government may introduce income taxation in order to effect the distribution of income and may possibly equally redistribute the income in more equal manner, with the help of different social services. However, on the other hand, incentive to supply labor force is highly influenced by the implementation of income tax, especially if it is in a progressive rate that is with higher marginal rate than average rate of tax. By reviewing proper literature, it becomes quiet evident that how these two contradicting concepts and trade-off between them effect the optimal income taxation theory. The analysis that is done by Mirrlees (1971) captured both equity and efficiency objectives while designing optimal income tax structure. More specifically, he introduced nonlinear structure of income taxation that might result in optimal conditions. According to his results, with an eye to meet equality objective income taxation cannot be in progressive rate. Besides, he also argued that supply of labor also need to be based on the ability of the workers, for example forcing certain group, with lowest ability, of labor not to work also might be optimal choice. An important assumption to include here is that both consumption and pre-tax income must be rising function of ability, for which all the results are based. In order to explore detailed information regarding the structure of taxation, it necessitates to take numerical analysis into consideration, which is available in the chapter regarding income taxation of "Public economics" by Myles (1995). One of the possible limitation of the Mirrlees economy in which he analyzed optimal income taxation is that he considered only one form of labor service. There are some other papers, such as Feldstein

(1973), in the literature, which makes design of optimal income tax structure more difficult by introducing more than one form of labor supply.

Even though, almost all of the paper in the literature regarding optimal income taxation assumes that there needs to be some part of government revenue, which is required to be collected by income taxation. Majority of the papers consider that this share of revenue as being constant number and to be exogenous. However, if we take various macroeconomic factors into account, it turns out that there should be some factors affecting total income tax revenue. Similar to the case of commodity taxation, there is a gap in the literature concerning possible determinants of income tax revenue. Specifically, what are the main determinants that dictate about the amount of income tax revenue? Furthermore, based on what criteria optimizing problem needs to be designed. What government function or objectives need to be taken into account while calculating optimal level of income tax revenue? In other words, what we need to optimize in order to get optimal level of income tax revenue? Such kind of question increased my incentives to come up with a mathematical model that can calculate the optimal level of income tax revenue, which will be analyzed in further parts of the paper. A point that can be mentioned here is that, the model tries to maximize long-run consumption, which is explained in the golden rule of Solow model of economic growth, by optimally calculating the level of different types of tax rates.

2.3. Combined optimal taxation

Finally, turning to the possible tax structure that combines both commodity tax and personal income tax, and simultaneously results in an optimal outcome for society, have been analyzed by several scientists, namely Revesz (1986), Mirrlees (1976) and Atkinson and Stiglitz (1976). Specifically, they based their work on the analysis of Mirrlees (1971) regarding optimal income taxation, by adding possible effect of commodity taxation on the Mirrlees economy. To be more precise, the study based on utility maximization problem considering both consumption of goods and labor supply (more clearly non-labor working time, which is leisure time) subject to income constraint, price of various types of commodities as well as ability of workforce. In that manner having leisure time also considered as being one of the commodities with its price being equal to hourly wage of the labor, since one needs to forgive one hour salary in order to get extra hour of leisure time. According to the claim of Atkinson and Stiglitz (1976)

commodity taxes can be equal to zero and implementation of only income tax is enough to meet welfare maximizing objective. Specifically, the results are based on the taxation of innate ability of individuals, however in the condition of separability, insignificant effect of commodity taxation on consumption choice is revealed. Alternatively, it also can be interpreted that the certain group of commodities that are being preferred more compared to other types of goods by consumers, who are supplying more working hours, needs to be taxed more. This is mainly due to the fact that there should be positive correlation between marginal rate of substitution of a commodity and labor as labor supply rises, ceteris paribus. Moreover, Mirrlees in his work in 1976, made above conclusion more stronger by emphasizing that the certain type of goods that are being more preferred by those workers with highest ability should be taxes the most with the highest rates. Alternatively, Christiansen (1984) made slightly different approach with respect to combined optimal taxation structure by considering income taxation and commodity taxation jointly. Specifically, he explains his structure in two stages. First step is the economy with the optimal income taxation structure without any commodity taxes. At the second stage, he includes commodity taxation and evaluates its effect on the welfare of households by keeping the tax revenue constant. According to his conclusions, it is possible to argue that positive commodity tax should be implemented on the goods, which demand on them increases as leisure time rises, but holding the income constant. Furthermore, he also claims that if change in leisure time do not effect on demand of the commodity, the tax on this particular commodity should be zero, again considering constant income. Finally, his third conclusion is that there should be negative commodity tax, if demand for the good falls by increase in leisure time, with the assumption of constant income.

As in the previous two cases, where commodity taxation and income taxation analyzed solely, in this combined optimal taxation structure also exists some limitations, even though it can solve for certain problems that was impossible with solely optimized tax structures. To be more specific, keeping the total revenue constant as in the case of Christiansen (1984), can balance the rates of personal income tax and consumption tax. However, his assumption of constant income makes it difficult to reveal appropriate taxation structure, since one of the main reasons of modifying labor supply, or leisure time as Christiansen stated, is the wage rate, which is also main source of income and it changes over time depending on the productivity of labor force.

Even though if it is possible to design optimal combined tax structure based on the model that was introduced by Christiansen, there is no clear information regarding total tax revenue. The same questions arise here as well regarding importance of macroeconomic factor dynamics and determinants of total tax revenue along with shares of each type of tax in the total revenue. Hence, as it emphasized above current paper tries to fill this gap in the literature by introducing a mathematical model that can reveal optimal levels of different types of taxes as well as it provides with an opportunity of balancing these optimal rates with respect to each other.

3. Mathematical model

In this part of the paper, we introduce a new mathematical model that calculates golden rate of the personal income tax rate and consumption tax rate. To be more specific, the mathematical model was developed based on the idea of golden rule of saving of basic Solow model of economic growth and basic consumption/saving theory of macroeconomics. In particular, according to basic consumption /saving theory, households spend certain portion of their income on consumption and the remaining part is saved. On the other hand, based on basic Solow model of economic growth, especially its golden rule of saving, in order to maximize long-run aggregate consumption saving rate needs to be equal to share of capital in the process of production (that is alpha from Cobb-Douglas production function $Y=A*K^{\alpha}*L^{1-\alpha}$). I included different types of taxes into the consumption/saving model and denoted rate of saving in terms of rate of consumption and tax rates. Precisely, households do not spend their income only on consumption, they also pay different types of taxes, such as personal income tax (directly) and consumption tax (indirectly). Hence, there is a possibility of revealing golden rate of different taxes as well, since golden rate of saving is already known. Generally the formulas for calculating golden rates of consumption tax and personal income tax are as follows:

$$b = \frac{i^p*(1-k-f^i-c^g-i^g)-(\alpha-i^g-f^i)*(c^p+i^p)}{c^p*(\delta_i*(1-k-f^i-c^g-i^g)-(\delta_c+\delta_i)*(\alpha-i^g-f^i))}$$

Where,

b – golden rate of consumption tax

i^p - private marginal propensity to invest

k – exports of goods and services as a percentage of revenue

f^i – foreign direct investment inflows as a percentage of revenue

c^g – government consumption as a percentage of revenue

i^g – government investment as a percentage of revenue

α – share of physical capital in production (from Cobb-Douglas production function)

c^p – private marginal propensity to consume

δ_i – effect of consumption tax on private investment

δ_c – effect of consumption tax on private consumption

$$a = 1 - x - \frac{\alpha - i^g - f^i}{(i^p - \delta_i * b * c^p) * (q + c^f * e + w * e + p * t)}$$

Where,

a – golden rate of personal income tax

x – other taxes of households as a percentage of income

α – share of physical capital in production (from Cobb-Douglas production function)

i^g – government investment as a percentage of revenue

f^i – foreign direct investment inflows as a percentage of revenue

i^p - private marginal propensity to invest

δ_i – effect of consumption tax on private investment

b – golden rate of consumption tax

q – remittances received as a percentage of revenue

c^f – consumption of an average firm as a percentage of firm's expenses

e – expenses of an average firm as a percentage of revenue

w – wages for employees as a percentage of expenses

p – profit of an average firm as a percentage of taxable income

t – taxable income of an average firm as a percentage of revenue

These are the final versions of the formulas. The derivation of the formulas could be found in https://docs.google.com/document/d/1nl-KIPug1EOOpHbHqUrv0fDTgHqGiib-/edit. In the following paragraph we try to test the applicability of the formulas by utilizing real life data for the case of Uzbekistan.

4. Interpretation of the results

Table 4: main findings (1) for non-optimal economy

	non-optimal economy

Year	Consumption tax rate (VAT+EXT)[9], without reinvestment	Consumption tax rate (VAT+EXT), with reinvestment	Consumption tax rate (only VAT), without reinvestment	Consumption tax rate (only VAT), with reinvestment	Consumption tax rate (only EXT), without reinvestment	Consumption tax rate (only EXT), with reinvestment
2010	-0.01	-0.02	-0.01	-0.02	0.00	-0.01
2011	0.11	0.09	0.07	0.06	0.04	0.03
2012	0.16	0.15	0.11	0.10	0.06	0.05
2013	0.12	0.11	0.08	0.07	0.04	0.04
2014	0.07	0.06	0.04	0.04	0.02	0.02
2015	-0.07	-0.09	-0.05	-0.06	-0.02	-0.03
2016	-0.12	-0.13	-0.08	-0.09	-0.04	-0.05
2017	-0.06	-0.07	-0.04	-0.05	-0.02	-0.02
2018	0.38	0.38	0.28	0.28	0.10	0.10
2019	0.24	0.24	0.18	0.18	0.06	0.06
average	**0.08**	**0.07**	**0.06**	**0.05**	**0.02**	**0.02**

Table 5: main findings (2) for optimal economy

	optimal economy					
Year	Consumption tax rate (VAT+EXT), without reinvestment	Consumption tax rate (VAT+EXT), with reinvestment	Consumption tax rate (only VAT), without reinvestment	Consumption tax rate (only VAT), with reinvestment	Consumption tax rate (only EXT), without reinvestment	Consumption tax rate (only EXT), with reinvestment
2010	0.02	0.01	0.02	0.01	0.01	0.00
2011	0.08	0.07	0.05	0.04	0.03	0.02
2012	0.13	0.11	0.08	0.07	0.04	0.04
2013	0.14	0.13	0.09	0.08	0.05	0.05
2014	0.11	0.10	0.07	0.07	0.04	0.04
2015	0.16	0.15	0.10	0.10	0.05	0.05
2016	0.14	0.13	0.09	0.08	0.05	0.04
2017	0.08	0.07	0.05	0.04	0.03	0.02
2018	0.04	0.03	0.03	0.03	0.01	0.01
2019	0.30	0.30	0.23	0.23	0.07	0.07
average	**0.12**	**0.11**	**0.08**	**0.08**	**0.04**	**0.03**

Above two tables mainly summarized the calculations of the golden rates of personal income tax and consumption tax rates for optimal and non-optimal economy. To be more specific, optimal economy is defined as when alpha in Cobb-Douglas production, which is share of capital stock in the process of production, equals marginal propensity to save. However, while developing mathematical model we split marginal propensity to save from basic

[9] VAT – value added tax
EXT – excise tax

consumption/saving theory. In particular, not all the saved money are invested, hence we introduced another variable that calculates only marginal propensity to invest. Table 5 represents golden rates of both consumption tax and personal income tax assuming that alpha (share of capital stock in the process of production) is equal to marginal propensity to invest. However, in the Table 4 for alpha was calculated by dividing private sector's expenditure on capital stock to the summation of private sector's expenditure on labor and capital stock. Alternatively, what percentage of private sector's expenditure is allocated to capital stock is represented by alpha. Besides, an important fact to include here is that according to the tax code of Uzbekistan there are two types of taxes, whose base is consumption of goods and services. To be more precise, value added tax and excise taxes are levied on the consumption of different types of commodities. Hence, we tried to calculate golden rate of these taxes both separately and together. Another grouping of the results is based on the fact that firms in private sector may keep certain portion of their profit rather than spreading all of it among shareholders in the form of dividends. Alternatively speaking, some of the firms may decide to reinvest particular portion of its profit in order to improve and enlarge the sales of the firm. Hence, different golden rates of consumption tax also calculated considering the aforementioned possibility. Turning to the results it is possible to emphasize that the economy in the previous decade was not in its optimal condition, which is natural for developing economy. Hence, lower consumption tax rate was required in order to support economic growth. As a matter of fact, there was 15 percent of value added tax on all products and services and excise tax on top of it. Specifically, rate of excise tax is different depending on the type of goods and services. In the model we just calculated the percentage share of excise tax in total consumption of goods and services. Based on the mathematical model that was developed in the previous part of the dissertation, it is possible to argue that consumption tax including both value added tax and excise tax needed to be equal to 7 percent to push the economic growth to its optimal level. On the other hand, personal income tax needed to be equal to 14 percent if assume that firms decide to reinvest particular portion of their profit. If we look at the key mathematical formulas for calculating golden rates of consumption tax and personal income tax, we may realize that calculations was based on (1) Private marginal propensity to consume, (2) Export of goods and services, (3) Foreign direct investment inflows, (4) Government's marginal propensity to

consume, (5) Government's marginal propensity to invest, (6) Share of capital stock in the production process, (7) Private marginal propensity to invest, (8) Remittances received by households, (9) Firms' consumption expenditure, (10) General expenses of the firms, (11) Average salary in the economy, (12) Profitability ratio in the private sector, (13) Corporate tax rate. Hence, it is believed that the model is performing well in terms of predicting different rates of taxes. Moreover, if we consider that none of the firms in the private sector decides to reinvest their profit, golden rate of consumption tax would be equal to 8 percent. Meaning that government need to penalize less for consuming. Or in different words, since any type consumption tax has a positive relationship with aggregate price level, decreasing consumption tax may shift aggregate demand curve upward by lowering aggregate price level. Basically, all calculations mainly based on marginal propensity to invest of both households and government. For instance, if the households marginal propensity to invest decreases consumption tax tends to increase, while personal income tax decreases. This is mainly due to the fact that, effect of personal income tax and consumption tax on households consumption/saving decision is different. Specifically, personal income tax effects on the general level of consumption, while consumption tax influences on the growth rate of consumption. For example, if we consider the consumption as an increasing function of income, personal income tax modifies the intercept of the curve, while consumption tax impacts on the slope. That is why, effect of consumption tax on aggregate consumption level is more strict and significant compared to the effect of personal income tax. Besides, the model also gives an opportunity to separate total consumption tax into two parts, namely value added tax and excise tax. In particular, out of this 8 percent how much of it needs to be collected through value added tax and excise tax. Precisely, the model predicts it as 6 to 2 respectively.

Now it is possible to turn to the Table 5, which mainly summarizes the results regarding golden rates of personal income tax and consumption tax in the case of optimal economy. According to the basic Solow model of economic growth if the aggregate investment rate in the economy reaches the level of alpha (share of capital stock in the process of production), economy will be in its optimal level. Optimal economy is defined as the aggregate consumption reaches its maximum level[10]. We artificially made investment rate equal to alpha, and tried to predict

[10] For detailed explanation see golden rule of saving of basic Solow model of economic growth

golden rate of tax rates. According to the results, consumption tax needs to be equal to 12 percent, while it is recommended to set personal income tax rate at 11 percent, for the condition if firms do not decide to reinvest. Besides, it is also possible to predict that two components of consumption tax separately. In particular, value added tax is recommended to be equal to 8 percent, while excise tax is the remaining part of the consumption tax, which is 4 percent on average. As we know excise tax is different depending on the type of goods and services, the model only can predict weighted average rate of excise tax. However, if some of the firms in private sector decides to reinvest certain part of their profit, the results will be slightly different. To be more specific, in that case consumption tax is required to be equal to 11 percent, while personal income tax needs to be set at 12 percent. This is mainly due to the fact that aforementioned level and growth effects of personal income tax and consumption tax on aggregate demand in the economy, which is one of the best proxies of economic growth and prosperity.

Now turning to the limitations of the model it is obvious from the tables that model is predicting certain negative values as well, which is impossible by nature. This might be the case due to the fact that the model is not in its perfect form. To be more specific, the mathematical model is still enjoying certain strict assumption, such as there is no tax avoidance and tax evasion, there is a constant returns to scale in the production process dividends, remittances and wages are considered as an income and taxed at the same level, there is no financial sector in the economy and many others. Indeed, the data is taken from the real world, in which none of these above assumptions hold. Besides, it is important to acknowledge that the model predicts optimal level of tax rates considering only economic growth as the main goal of economic policymaking. However, there are other functions of government from levying taxes. However, in general, it is an undisputable fact that the tax system is such a system that cannot be reformed annually. In particular, it is difficult to adjust the tax system to the macroeconomic fluctuations so often. Normally, tax reforms take place in every decade. Considering this feature of the taxation, it is possible to argue that the model is performing well enough to recommend some policy implications.

5. Conclusions and recommendations

To conclude with it is important to restate that the dissertation was mainly concentrated on contributing the optimal taxation theory by developing a new mathematical model that can predict golden rates of different types of taxes, such as personal income tax and consumption tax. Generally, the model was based on golden rule of saving of basic Solow model of economic growth and basic consumption/saving theory of macroeconomics. Appropriateness of the mathematical model was evaluated considering the case of Uzbekistan, and it is performing well in terms of prediction. However, the model still can be developed further by properly adjusting it to above stated assumptions. Besides, there might be some more assumptions as well that are not included in the list and the model possibly may be enjoying them as well. Hence, the list of assumptions also needs to be reviewed and enlarged if necessary in order to increase the appropriateness of the model. Finally, it is recommended to set tax rates according to the model that was initiated in the paper, which takes different important macroeconomic indicators into account while predicting golden rates of various types of taxes.

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