VEKTOR MAYDONIDAGI BIRINCHI TARTIBLI AMALLARNI GAMILTON OPERATORI BILAN ALMASHTIRISH

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ANNOTATSIYA: Bu ishda vektor analizning asosiy tushunchalaridan biri Gamilton operatori haqida ma’lumot keltirilgan. Birinchi tartibli differensial vektor amallar keltirib o’tgan.

Kalit so’z: Gamilton operatori, vektor, gradient, divergensiya

FIRST-ORDER OPERATIONS IN THE VECTOR FIELD EXCHANGE WITH THE HAMILTON OPERATOR

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ABSTRACT: In this work, one of the main concepts of vector analysis is information about the Hamiltonian operator. The differential vector of the first order was cited by the operations.

Keywords: Hamilton operator, vector, gradient, divergence

ОПЕРАЦИИ ПЕРВОГО ПОРЯДКА В ВЕКТОРНОМ ПОЛЕ ОБМЕН С ОПЕРАТОРОМ ГАМИЛЬТОНА

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АННОТАЦИЯ: В этой работе представлена информация об одном из основных понятий векторного анализа-операторе Гамильтона. Дифференциальные векторные операции первого порядка приведены.

Ключевые слова: Оператор Гамильтона, вектор, градиент, расходимость.

$Oxyz$ fazoning ω sohasida

$$\vec{a}(M)=P(x,y,z)\vec{i}+Q(x,y,z)\vec{j}+R(x,y,z)\vec{k}$$

vektor maydon berilgan bo‘lsin, unda $P(x,y,z), Q(x,y,z), R(x,y,z)$ funksiyalar differensiallanuvchi funksiyalar.

Divergensiyani hisoblashda quyidagi xossalardan foydalaniladi:

$1^0 . \div \left(\vec{a}(M)+\vec{b}(M)\right)=¿\vec{a}(M)+¿\vec{b}(M);$

$2^0 . divC\cdot\vec{a}(M)=C\cdot\div\vec{a}(M), bunda C-o‘ zgarmas son$

$3^0 . divu(M)\cdot\vec{a}(M)=u(M)\div\vec{a}(M)+\vec{a}(M)\, grad\, u(M),$

bu yerda $u(M)-¿$skalyar maydonni aniqlovchi funksiya.

Ta’rif. $\vec{a}(M)$ vektor maydonning *diverginsiyasi (uzoqlashuvchisi)* deb M nuqtaning skalyar maydoniga aytiladi, u $¿\vec{a}(M)$ ko‘rinishda yoiladi va

$$¿\vec{a}(M)=\frac{\partial P}{\partial x}+\frac{\partial Q}{\partial y}+\frac{\partial R}{\partial z}(1)$$

formula bilan aniqlanadi, bu yerda xususiy hosilalar M nuqtada hisoblanadi.

Agar fazodagi biror D soxaning xar bir $M=M(x,u,z)$ nuqtasida $u=u(M)=f(x,u,g)$skalyar funksiya berilgan bo’lsa, u xolda bu soxada skalyar maydon berilgan deyiladi. $u=f(x,u,z)$ funktsiya maydon funksiyasi deyiladi.

Faraz qilaylik, $Oxyz$ fazoning ω sohasida quyidagi vektor maydon berilgan bo‘lsin:

$$\vec{a}(M)=P(x,y,z)\vec{i}+Q(x,y,z)\vec{j}+R(x,y,z)\vec{k}.$$

Ta’rif. $\vec{a}(M)$ vektor maydonning *uyurmasi* (yoki *rotori*) deb M nuqtaning $rot\,\vec{a}(M)$ bilan belgilanadigan va

$$rot\,\vec{a}(M)=\left(\frac{\partial R}{\partial y}-\frac{\partial Q}{\partial z}\right)\vec{i}+\left(\frac{\partial P}{\partial z}-\frac{\partial R}{\partial x}\right)\vec{j}+\left(\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}\right)\vec{k}(2)$$

formula bilan aniqlanadigan vektor maydoniga aytiladi, bunda xususiy hosilalarni $M(x,y,z)$ nuqtada topamiz.

Uyurmaning ta’rifidan foydalanib, quyidagi xossalarning to‘g‘ri ekaniga ishonch hosil qilish mumkin:

$1^0 . rot\left(\vec{a}+\vec{b}\right)=rot\,\vec{a}+rot\,\vec{b};$

$2^0 . rot\left(C\vec{a}\right)=Crot\,\vec{a}$,bunda $C-¿$o‘zgarmas skalyar;

$3^0 . rot(u\vec{a})=u\cdot rot\,\vec{a}+(grad\,u)\times\vec{a},$ bunda $u=u(M)$ skalyar maydonni aniqlovchi funksiya.

Vektor analizning $grad,\div,rot$ differensial amallarini simvolik ∇ vektor yordamida (Nabla vektor-Gamilton operatori) ifodalash qulaydir:

$$\nabla = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial x}\vec{j} + \frac{\partial}{\partial x}\vec{k}.$$

Bu vektorni u yoki bu (skalyar yoki vektor) kattalikka qo‘llanishni bunday tushunmoq kerak: vektor algebra qoidalariga ko‘ra bu vektorni berilgan kattalikka ko‘paytirish amalini bajarish lozim, so‘ngra $\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}$ simvollarning bu kattalikka ko‘paytirishni tegishli hosilani topish sifatida qarash kerak.

Bu vektor bilan amallar bajarish qoidalarini qarab chiqamiz:

1. ∇ nabla-vektorning $u(M)$ skalyar funksiyaga ko‘paytmasi shu funksiyaning gradientini beradi:

$$\nabla u = \left(\frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}\right)u = \frac{\partial u}{\partial x}\vec{i} + \frac{\partial u}{\partial y}\vec{j} + \frac{\partial u}{\partial z}\vec{k} = grad\, u.$$

Shunday qilib, $\nabla u = grad\, u$.

2. ∇ nabla-vektorning

$$\vec{a}(M) = P(x, y, z)\vec{i} + Q(x, y, z)\vec{j} + R(x, y, z)\vec{k}$$

vektor funksiya bilan skalyar ko‘paytmasi shu funksiyaning divergensiyasini beradi:

$$\nabla \cdot \vec{a} = \left(\frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}\right)\cdot\left(P(x, y, z)\vec{i} + Q(x, y, z)\vec{j} + R(x, y, z)\vec{k}\right) = ¿$$

$$¿\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} = ¿\vec{a}.$$

Shunday qilib, $\nabla \cdot \vec{a} = ¿\vec{a}$.

3. ∇ nabla-vektorning

$$\vec{a}(M) = P(x, y, z)\vec{i} + Q(x, y, z)\vec{j} + R(x, y, z)\vec{k}$$

vektor funksiyaga vektor ko‘paytmasi shu funksiyaning uyurmasini beradi:

$$\nabla \times \vec{a} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix} = ¿$$

$$¿\left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right)\vec{i} + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right)\vec{j} + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\vec{k} = rot\, \vec{a}.$$

Shunday qilib, $\nabla \times \vec{a} = rot\, \vec{a}$.

Vektor maydondagi ikkinchi tartibli amallarni ko‘ramiz. Shuni aytib o‘tish kerakki, $grad u, rot \vec{a}$ amallari vektor maydonlarni vujudga keltiradi, $¿\vec{a}$ amali esa skalyar maydonni vujudga keltiradi. ko‘rsatilgan amallarning quyidagi kombinatsiyalari bo‘lishi mumkin: $¿ grad u, grad \div \vec{a}, rot\, rot\, \vec{a}, \div rot \vec{a}$, bular ikkinchi tartibli amallar deyiladi. Ulardan eng muhimlarini qarab chiqamiz.

1.$¿ rot \vec{a} = 0$.

Haqiqatan ham, agar vektor maydon

$$\vec{a} = P(x, y, z)\vec{i} + Q(x, y, z)\vec{j} + R(x, y, z)\vec{k}$$

bo‘lsa, u holda ikkinchi tartibli aralash hosilalarning tengligi uchun

$$¿ rot \vec{a} = \frac{\partial}{\partial x}\left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right) + \frac{\partial}{\partial x}\left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right) + \frac{\partial}{\partial x}\left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) = ¿$$

$$¿ \frac{\partial^2 R}{\partial x \partial y} - \frac{\partial^2 Q}{\partial x \partial z} + \frac{\partial^2 P}{\partial y \partial z} - \frac{\partial^2 R}{\partial y \partial x} + \frac{\partial^2 Q}{\partial x \partial z} - \frac{\partial^2 P}{\partial z \partial y} = 0$$

bo‘ladi. Shu natijaning o‘zini nabla-operator

$$¿ rot \vec{a} = \nabla \cdot (\nabla \times \vec{a})$$

yordamida ham olish mumkin, chunki bu yerda uchta vektorning aralash ko‘paytmasini hosil qilamiz: ∇, ∇ va $\vec{a}$, bularning ikkitasi bir xil. Bunday ko‘paytma nolga teng bo‘lishi ravshan.

2.$rot\, grad u = 0$.

Haqiqatan,

$$grad u = \frac{\partial u}{\partial x}\vec{i} + \frac{\partial u}{\partial y}\vec{j} + \frac{\partial u}{\partial z}\vec{k}$$

bo‘lgani uchun ikkinchi tartibli aralash ko‘paytmalarning tengligi tufayli:

$$rot\, grad u = \vec{i}\left[\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial z}\right) - \frac{\partial}{\partial z}\left(\frac{\partial u}{\partial y}\right)\right] + \vec{j}\left[\frac{\partial}{\partial z}\left(\frac{\partial u}{\partial x}\right) - \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial z}\right)\right] + ¿$$

$$+\vec{k}\left[\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right) - \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}\right)\right] = \vec{i}\left(\frac{\partial^2 u}{\partial y \partial z} - \frac{\partial^2 u}{\partial z \partial y}\right) + \vec{j}\left(\frac{\partial^2 u}{\partial z \partial x} - \frac{\partial^2 u}{\partial x \partial z}\right) + ¿$$

$$+\vec{i}\left(\frac{\partial^2 u}{\partial x \partial y} - \frac{\partial^2 u}{\partial y \partial x}\right) = \vec{0}.$$

Shu natijaning o‘zini ∇ nabla-operator yordamida ham hosil qilish mumkin:

$$rot\, grad u = \nabla \times \nabla u = (\nabla \times \nabla) u = \vec{0},$$

chunki bir xil vektorlarning vektor ko‘paytmasi nol vektorga teng.

$3.¿\, grad\, u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}.$

Haqiqatan ham,

$$grad\, u = \frac{\partial u}{\partial x}\vec{i} + \frac{\partial u}{\partial y}\vec{j} + \frac{\partial u}{\partial z}\vec{k}$$

bo‘lgani uchun

$$¿\, grad\, u = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{\partial u}{\partial z}\right) = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \quad (3)$$

bo‘ladi.

(3) tenglikning o‘ng tomoni simvolik tarzda bunday belgilanadi:

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$$

yoki

$$\Delta u = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) u.$$

Bunda

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \quad (4)$$

simvol *Laplas operatori* deyiladi. Bu operatorni ∇ vektorning skalyar kvadrati tarzida qarash tabiiydir. Gamilton operatorining skalar maydoni Laplas operatorini beradi.

Adabiyotlar

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