# USING GEOGEBRA TO CALCULATE DOUBLE INTEGRALS.

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**Abstract:** This article briefly discusses the analysis and calculation of multiple integrals using the GeoGebra program when calculating a double integral, setting a limit over a given domain, and changing variables.

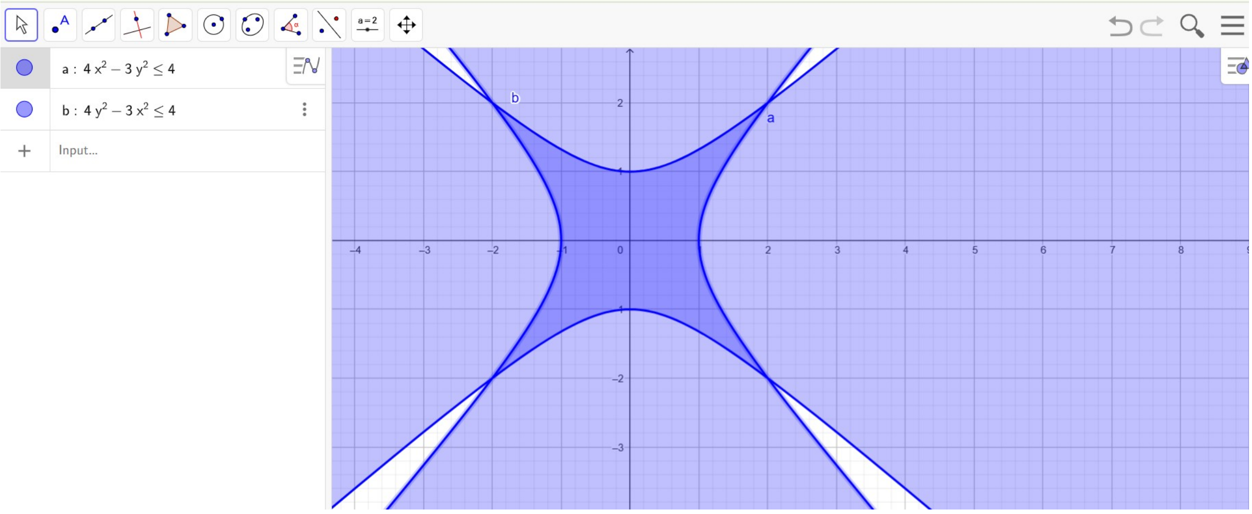
**Keywords:** double integral, closed domain, entering a variable into a double integral, GeoGebra program.

We know from mathematical analysis or higher mathematics, which are the subjects of mathematics, physics, engineering, economics and other areas of higher education, that the concept of a definite integral of a multivariable function leads to the concept of double or triple, ... integrals. When finding the multiple integral of a function through a form bounded by a closed domain in a certain interval, it is necessary to know the appearance of the domain and to know the exact image of the domain in order to determine the intervals of change of variables. [4]

Below, we will use the Geogebra program to conclude that when calculating the multiple integral using domains bounded by the graphs of a certain function, it is necessary to set limits on variables or use the method of introducing variables in the multiple integral.

Example 1: $\iint (x^3 y + x y^3)\, dxdy$ Calculate the double integral in the domain given below. $D = \{(x, y) \in R^2, x \geq 0, y \geq 0, 4x^2 - 3y^2 \leq 4, 4y^2 - 3x^2 \leq 4\}$ [1,2]

**Solution:** Before calculating this multiple integral, let's first define the shape of the sphere. We will use the GeoGebra program to do this.




Since both variables in the domain are positive, the first quarter of the coordinate plane of the domain is taken. Now, since it is somewhat difficult to set a limit in this domain, we use the method of introducing variables in the multiple integral. For this:

$\begin{cases} x = u\,ch\,v \\ y = \dfrac{2}{\sqrt{3}} u\,sh\,v \end{cases}$ We can introduce variables such as . Now we define the boundaries of the new variables. To do this, first $4x^2 - 3y^2 \le 4$ by putting variables into the equation $0 \le u \le 1$ we find that is. Now $4y^2 - 3x^2 \le 4$ from inequality $0 \le sh\,v \le \sqrt{3}$ we find that.

When changing variables in a multiple integral, it is necessary to find the Yakabian, and the Yakabian is

$I(u,v) = \begin{vmatrix} x'_u & x'_v \\ y'_u & y'_v \end{vmatrix} \ne 0$ is found using the formula [1,2,3].

So now we find the Jacobian in terms of the variables introduced above.

$\begin{vmatrix} x'_u & x'_v \\ y'_u & y'_v \end{vmatrix} = \begin{vmatrix} chv & ushv \\ \frac{2}{\sqrt{3}} shv & \frac{2}{\sqrt{3}} uchv \end{vmatrix} = \frac{2}{\sqrt{3}} u$ is equal to . Now we calculate the value of the integral by putting all the values found into the integral. The integral

$$\iint (x^3 y + x y^3) dxdy = \int_0^1 du \int_0^{sh^{-1}\sqrt{3}} \frac{4}{3} u^5 chv\, shv (ch^2 v + \frac{4}{3} sh^2 v) dv$$

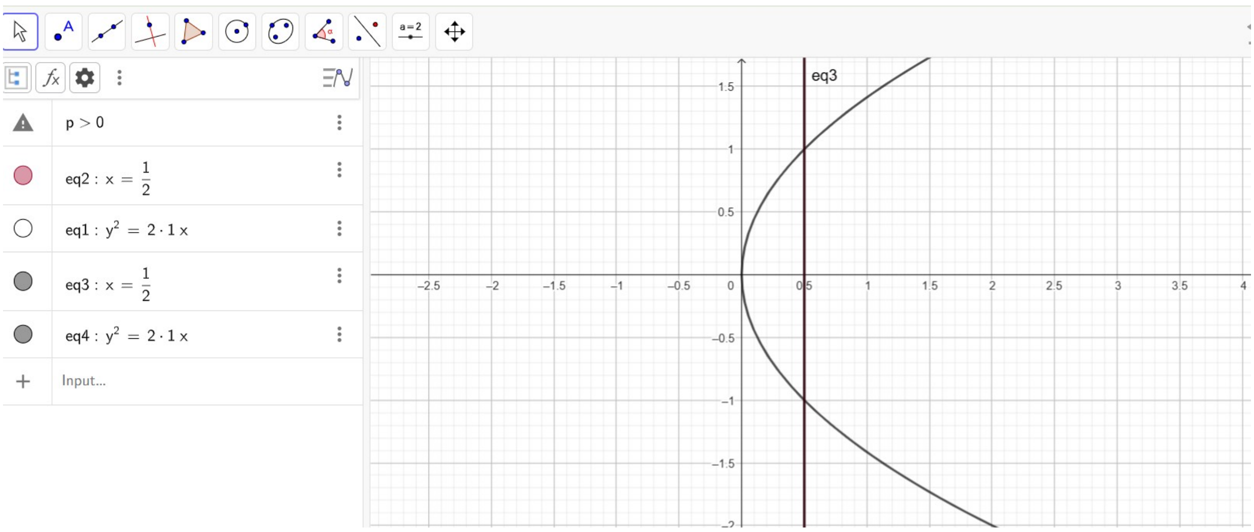
appears (here $sh^{-1}v$it's shv (the inverse function of the function) . Now $ch^2 v = 1 + sh^2 v$ using the fact that

$$\int_0^1 du \int_0^{sh^{-1}\sqrt{3}} \frac{4}{3} u^5 chv\, shv (ch^2 v + \frac{4}{3} sh^2 v) dv = \int_0^1 du \int_0^{sh^{-1}\sqrt{3}} \frac{4}{3} u^5 shv (1 + \frac{7}{3} sh^2 v) d(shv)$$

If we calculate the integral by expressing it in the form $\iint (x^3 y + x y^3) dxdy = \frac{3}{2}$ It turns out that.

Example 2: $\iint x y^2 dxdy$ multiple integral, $D = \{(x, y) \in R^2, y^2 = 2px, x = \frac{p}{2}, p > 0\}$ Calculate in the field. [1,2]

Solution: As above, we first draw the domain in Geogebra to determine the limits of the integral.



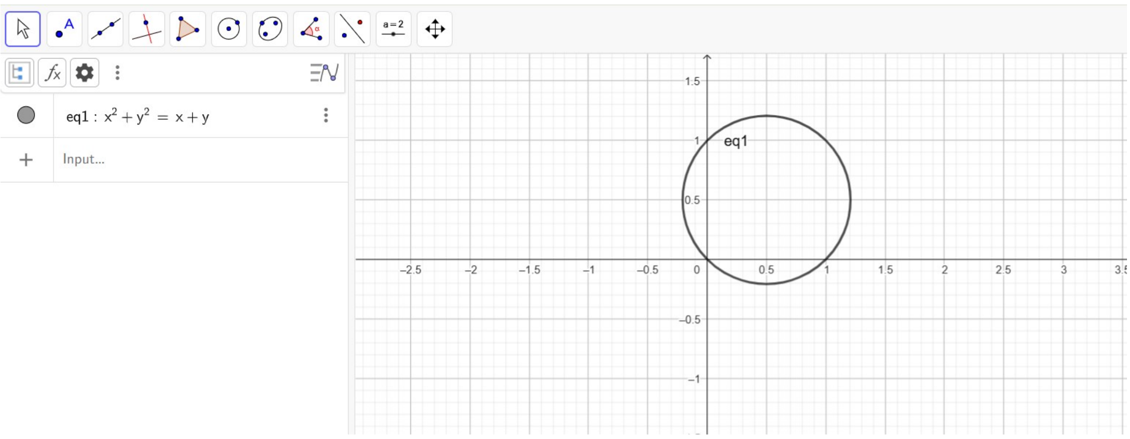
So the domain is a closed domain bounded by lines. Now we define the boundaries. From this, the boundaries of the variables $\{0 \le x \le \frac{p}{2}, -\sqrt{2px} \le y \le \sqrt{2px}\}$ It follows that. If we calculate the integral from this

$$\iint x y^2 dxdy = \int_0^{\frac{p}{2}} dx \int_{-\sqrt{2px}}^{\sqrt{2px}} x y^2 dy = \frac{p^5}{21}$$

It turns out that.

Example 3: $\iint (x+y) dxdy$ integral $D = \{(x, y) \in R^2, x^2 + y^2 = x + y\}$ Calculate in the field. [1,2]

Solution: Just like in the two examples above, to find the solution to this example, we will draw the sphere using Geogebra and




Now, in calculating this multiple integral, we also use the method of changing variables. That is, usually if the domain of the multiple integral is a circle or an ellipse, then it is very convenient to switch to polar coordinates in such a domain. Therefore, we use polar substitution. To do this, we first write down the full form of the equation of the circle from the given domain.

$x^2 + y^2 - x - y = 0 \rightarrow \left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{1}{2}$ and we introduce the following definitions here.

$\begin{cases} x-0.5=rcos\varphi \\ y-0.5=rsin\varphi \end{cases} \to \begin{cases} x=rcos\varphi+0.5 \\ y=rsin\varphi+0.5 \end{cases}$ here $0\le r\le \frac{1}{\sqrt{2}}, 0\le\varphi\le 2\pi$ We find the Yakabian just as in Example 1 above.

$$\begin{vmatrix} x'_r & x'_\varphi \\ y'_r & y'_\varphi \end{vmatrix} = \begin{vmatrix} cos\varphi & -rsin\varphi \\ sin\varphi & rcos\varphi \end{vmatrix} = r$$

Now we calculate the integral by setting all the limits.

$\int\limits_0^{\frac{1}{\sqrt{2}}} dr \int\limits_0^{2\pi} r\left(rcos\varphi+rsin\varphi+1\right)d\varphi=\frac{\pi}{2}$ It follows that it is equal to .

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