## KASR TARTIBLI DIFFERENSIAL OPERATOR ISHTIROK ETGAN INTEGRO-DIFFERENSIAL TENGLAMALAR UCHUN INTEGRAL SHARTLI MASALALAR

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**Annotatsiya**

Ushbu maqolada kasr tartibli differensial operator qatnashgan integro-differensial tenglama uchun ikkinchi tur integral shartli masala o‘rganilgan.

**Kalit so`zlar:** tenglama,chegaraviy shartlar, chegaralangan va bo‘lakli uzluksiz bo‘lgan ma’lum funksiyalar, uzluksiz ma’lum funksiyalar, ikkinchi tur Fredgolm integral tenglamasi

## FRACTIONAL DIFFERENTIAL OPERATOR PARTICIPATION FOR INTEGRO-DIFFERENTIAL EQUATIONS INTEGRAL CONDITIONAL PROBLEMS

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**Abstract**

In this article, the integral conditional problem of the second type for the integro-differential equation involving a fractional differential operator is studied.

**Key words:** equation, boundary conditions, certain bounded and piecewise continuous functions, certain continuous functions, Fredholm integral equation of the second kind

**1-masala.**

$$y''(x)+p_1(x)y'(x)+p_2(x)y(x)+p_3(x)D_{ax}^{\alpha}\omega(x)y(x)=f(x),$$

$$x\in(a,b) \qquad (1)$$

tenglamaning $[a,b]$ segmentda aniqlangan, uzluksiz va

$$y(a)=k_1,\;\; y'(b)+hy(b)=h\int_{\alpha}^{\beta}y(t)dt+k_2 \qquad (2)$$

shartlarni qanoatlantiruvchi yechimi topilsin, bu yerda $k_1,k_2,h,\alpha,\beta$-berilgan sonlar bo‘lib, $a\le\alpha<\beta\le b$.

(2) dan ko‘rinib turibdiki $h=0$ da 1-masaladan

$$y(a)=k_1 \qquad y'(b)=k_2 \qquad (3)$$

chegaraviy shartlarni qanoatlantiruvchi masala kelib chiqadi. Agar $0<|h|\le 1$ va $a<\alpha<\beta<b$ bo‘lsa, (2) shartlarning ikkinchisini undagi integralga o‘rta qiymat haqidagi teoremani tatbiq qilib, $y'(b)+hy(\xi)=k_2$ ko‘rinishda yozib olish mumkin bo‘ladi, bu yerda $\xi$ - $[a,b]$ segmentdagi qandaydir tayinlangan son. Demak, bu holda, 1-masala

$$y(a)=k_1, \qquad y'(b)+hy(\xi)=k_2 \qquad (4)$$

shartlarni qanoatlantiruvchi yechimi topilganidek o‘rganiladi. $0<|q|<1$ va $[\alpha,\beta]=[a,b]$ bo‘lgan holda ham 1-masala 4-shartga keltirib, o‘rganiladi.

Yuqoridagilarni e’tiborga olgan holda 1-masalani $h=1$, $\alpha=a$, $\beta=b$ bo‘lgan holda, ya’ni (2) shartlar

$$y(a)=k_1 \qquad y'(b)+y(b)=\int_a^b y(t)dt+k_2 \qquad (5)$$

ko‘rinishga ega bo‘lgan holda o‘rganamiz.

Bu masalaning yechimi mavjud va yagonaligini ko‘rsatish uchun xuddi 3-shartdagi kabi, (1) tenglamani $[a,x]$ oraliqda ikki marta integrallab,

$$y'(x)+p_1(x)y(x)+\int_a^x\left\{p_2(t)-p_1'(t)+\frac{\omega(t)}{\Gamma(1-\alpha)}\left[p_3(x)(x-t)^{-\alpha}-\right.\right.$$
$$\left.\left.-\int_t^x p_3'(z)(z-t)^{-\alpha}\,dz\right]\right\}y(t)dt=\int_a^x f(t)dt+y'(a)+k_1p_1(a), \qquad (6)$$

va

$$y(x)+\int_a^x\left\{p_1(t)+\left[p_2(t)-p_1'(t)\right](x-t)+\right.$$
$$\left.+\frac{\omega(t)}{\Gamma(1-\alpha)}\int_t^x(\xi-t)^{-\alpha}\left[p_3(\xi)-p'_3(\xi)(x-\xi)d\xi\right]y(t)dt\right\}= \qquad (7)$$
$$=\int_a^x(x-t)f(t)dt+y'(a)(x-a)+k_1p_1(a)(x-a)+k_1$$

tengliklarga ega bo‘lamiz. (6) va (7) tengliklarni quyidagicha yozib olamiz;

$$y(x)=-\int_a^x\left\{p_1(t)+\left[p_2(t)-p_1'(t)\right](x-t)+\right.$$
$$\left.+\frac{\omega(t)}{\Gamma(1-\alpha)}\int_t^x(\xi-t)^{-\alpha}\left[p_3(\xi)-p'_3(\xi)(x-\xi)d\xi\right]y(t)dt\right\}+$$
$$+\int_a^x(x-t)f(t)dt+y'(a)(x-a)+k_1p_1(a)(x-a)+k_1,$$

$$y'(x)=-p_1(x)y(x)-\int_a^x\left\{p_2(t)-p_1'(t)+\frac{\omega(t)}{\Gamma(1-\alpha)}\left[p_3(x)(x-t)^{-\alpha}-\right.\right.$$
$$\left.\left.-\int_t^x p_3'(z)(z-t)^{-\alpha}\,dz\right]\right\}y(t)dt+\int_a^x f(t)dt+y'(a)+k_1p_1(a).$$

Bulardan quyidagiga ega bo‘lamiz:

$$y(x)=\int_a^x K_2(x,t)y(t)dt+f_2(x)+y'(a)(x-a), \qquad (8)$$

$$y'(x)=-p_1(x)y(x)-\int_a^x K_1(x,t)y(t)dt+f_1(x)+y'(a), \quad (9)$$

bu yerda

$$f_1(x)=\int_a^x f(t)dt+k_1p_1(a), \qquad f_2(x)=\int_a^x (x-t)f(t)dt+k_1p_1(a)(x-a)+k_1,$$

$$K_1(x,t)=p_1(t)+[p_2(t)-p_1'(t)](x-t)+$$
$$+\frac{\omega(t)}{\Gamma(1-\alpha)}\int_t^x (\xi-t)^{-\alpha}\left[p_3(\xi)-p'_3(\xi)(x-\xi)d\xi\right],$$

$$K_2(x,t)=-p_1(t)+[p_2(t)-p_1'(t)](x-t)-$$
$$-\frac{\omega(t)}{\Gamma(1-\alpha)}\int(\xi-t)^{-\alpha}\left[p_3(\xi)-p'_3(\xi)(x-\xi)\right].$$

$K_1(x,t)$, $K_2(x,t)$–$\{(x,t): a\le x\le b\}$ to‘rtburchakda chegaralangan va bo‘lakli uzluksiz bo‘lgan ma’lum funksiyalar, $f_1(x)$, $f_2(x)$ esa $[a,b]$ da uzluksiz ma’lum funksiyalar.

(8) dan quyidagini topamiz:

$$\int_a^b y(t)dt=\int_a^b\int_a^b K_2(x,t)y(t)dt+\int_a^b f_2(t)dt+\frac{1}{2}y'(a)(b-a)^2.$$

Buni va

(8) va (9) tengliklarda $x=b$ deb,

$$y'(b)=-p_1(b)y(b)-\int_a^b K_1(b,t)y(t)dt+f_1(b)+y'(a), \quad (10)$$

$$y(b)=\int_a^b K_2(b,t)y(t)dt+f_2(b)+y'(a)(b-a), \quad (11)$$

tengliklarni topamiz. (10) va (11) ni (5) shartga qo‘yib,

$$y'(a)\cdot\left\{(b-a)\left[-p_1(b)+1\right]+1\right\}=$$
$$=-\left[-p_1(b)+1\right]\cdot\int_a^b K_2(b,t)y(t)dt+\int_a^b K_1(b,t)y(t)dt-$$
$$-f_2(b)\left[-p_1(b)+1\right]-f_1(b)+\int_a^b y(t)dt+k_2$$

tenglikka ega bo‘lamiz.

Agar $(b-a)\left[-p_1(b)+1\right]+1\neq 0$ bo‘lsa $y'(a)$ oxirgi tenglikdan bir qiymatli topiladi

$$y'(a)=\left\{-\left[-p_1(b)+1\right]\cdot\int_a^b K_2(b,t)y(t)dt+\int_a^b K_1(b,t)y(t)dt-\right.$$
$$\left.-f_2(b)\left[-p_1(b)+1\right]-f_1(b)+\int_a^b y(t)dt+k_2\right\}\cdot\frac{1}{(b-a)\left[-p_1(b)+1\right]+1}.$$

Uni (8) ga qo‘yib,

$$y(x)=\int_a^x K_2(x,t)y(t)dt+f_2(x)+\left\{-\left[-p_1(b)+1\right]\cdot\int_a^b K_2(b,t)y(t)dt+\int_a^b K_1(b,t)y(t)dt-\right.$$
$$\left.-f_2(b)\left[-p_1(b)+1\right]-f_1(b)+\int_a^b y(t)dt+k_2\right\}\cdot\frac{x-a}{(b-a)\left[-p_1(b)+1\right]+1} \qquad (12)$$

$y(x)$ ga nisbatan ikkinchi tur Fredgolm integral tenglamasiga ega bo‘lamiz.

Agar berilganlarga qo‘yilgan $f(x)\in C[a,b]$, $p_1(x)\in C^1[a,b]$, $p_2(x), p_3(x)\in C^2[a,b]$ shartlarda $\left|K_2(x,t)\right|<1$ bo‘lsa, (12) tenglama va, demak, 1-masala yagona yechimga ega bo‘ladi.

## Foydalanilgan adabiyotlar.


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