PLANIMETRIK MASALALARNI YECHISHDA TENGSIZLIKLARNI QO'LLASH USULLARI

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Annotatsiya: Ma'lumki, elementar matematika kursida geometriya fani shartli ravishta ikki qismga bo'linadi: planimetriya va stereometriya. Geometriya bo'yicha ba'zi masalalarni, masalan, eng katta yoki eng kichik miqdorlarni topishga oid masalalarni yechish uchun ko'p jihatdan ularni yechish tajribasiga va yechish usullarini o'zlashtirganlik darajasiga bog'liq. Shuning uchun berilgan qarab qaysi tengsizlikni qo'llash va uni isbotlash muhimdir. Endi quyida ba'zi masalalarni qarab chiqaylik.

Kalit so'zlar: Geometriya, Romb, tengsizlik, tranzitivlik, mediana, o'rtacha qiymat, o'rta arifmetik, parallelogramm, To'rtburchak.

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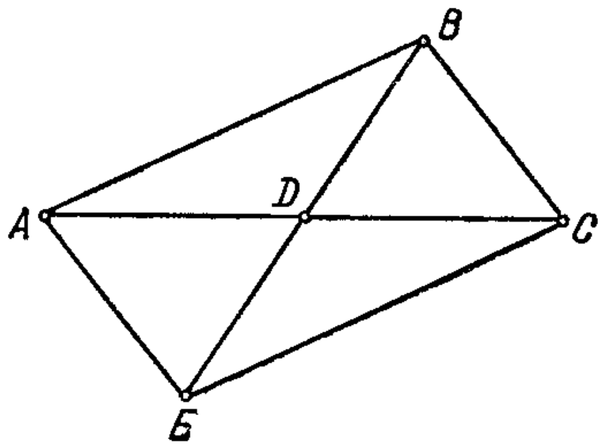
Annotation: As is known, in the elementary mathematics course, the subject of geometry is conventionally divided into two parts: planimetry and stereometry. Solving some problems in geometry, for example, problems related to finding the largest or smallest quantities, largely depends on the experience of solving them and the level of mastery of the solution methods. Therefore, it is important to apply which inequality to use and prove it depending on the given. Now let's look at some problems below.

Keywords: Geometry, Rhombus, inequality, transitivity, median, average value, arithmetic mean, parallelogram, Rectangle.

1-masala.Istalgan uchburchakda katta tomonga kichik mediana tushishini isbotlang.

Isbot. ABCuchburchakda $BC=a, AC=b, AB=c$va medianasi $BD=m_b$ bo'lsin (1-chizma).



(1-chizma)

BDmediananing davomidan DEkesmani, BDmedianaga teng qilib davom ettiramiz. To'rtburchak $ABCE-$¿parallelogramm, uning uchun diagonallari kesishish nuqtasida teng ikkiga bo'linadi. Geometriyadan ma'lumki, $AC^2+BE^2=2AB^2+2BC^2$ yoki bizning belgilashimiz bo'yicha:

$b^2+(2m_b)^2=2c^2+2a^2$.Bundan $m_b=\frac{\sqrt{2c^2+2a^2-b^2}}{2}$ni topamiz. Shunga o'xshash $m_a=\frac{\sqrt{2c^2+2b^2-a^2}}{2}$ni ham topamiz. Aniqlik uchun $a<b$bo'lsin. Unda $m_a>m_b$ kerak, ya'ni $\sqrt{2c^2+2b^2-a^2}>\sqrt{2c^2+2a^2-b^2}$yoki $2b^2-a^2>2a^2-b^2$ yoki $a^2<b^2$ va $a<b$. Biz $a<b$bo'lsa,$m_a>m_b$bo'lishini isbotladik.

2-masala. Rombning perimetrini diagonallarining yig'indisiga nisbati k bo'lsa, $\sqrt{2}<k<2$ munosabat o'rinli ekanligini isbotlang.

Isbot. Rombning tomoni $a$va o'tkir burchagini αdeb belgilaymiz (1-chizma). Rasmdan ko'rinib turibdiki: $\frac{AC}{2}=OC=a\sin\frac{\alpha}{2}$ va $AC=2a\sin\frac{\alpha}{2}$;

$\frac{BD}{2}=OD=a\cos\frac{\alpha}{2}$ va $BD=2a\cos\frac{\alpha}{2}$.

$$k=\frac{4a}{AC+BD}=\frac{4a}{2a\left(\sin\frac{\alpha}{2}+\cos\frac{\alpha}{2}\right)}=\frac{2}{\sqrt{2}\sin\left(\frac{\alpha}{2}+45°\right)} \qquad (1)$$

Bu yerda $0<\frac{\alpha}{2}<45°$ ekanligini hisobga olsak,

$$\frac{\sqrt{2}}{2}<\sin\left(\frac{\alpha}{2}+45°\right)\leq 1$$

(1) va (2) ifodalardan xulosa qilsak, $\sqrt{2}\leq k<2$ kelib chiqadi.

3-masala. ABCuchburchakda Mnuqta olingan. Bu nuqtadan AM, BM, CM to'g'ri chiziq o'tkazilgan. Bu to'g'ri chiziqlarning davomining uchburchak tomonlari bilan kesishish nuqtalari mos ravishda A_1, B_1, C_1bo'lsa, quyidagi tengsizlik bajarilishini ko'rsating:

$$\frac{AM}{A_1M}\frac{BM}{B_1M}\frac{CM}{C_1M}\geq 8$$

Isbot. Biz oldingi masalalardan birida ko'rgan tenglikdan foydalanamiz:

$$\frac{AM}{A_1M}=\frac{S_2+S_3}{S_1};\ \frac{BM}{B_1M}=\frac{S_1+S_3}{S_2};\ \frac{CM}{C_1M}=\frac{S_2+S_1}{S_3};$$

Bu yerda $S_1, S_2, S_3 - MBC, MCA, MAB$ uchburchaklarning yuzlari.

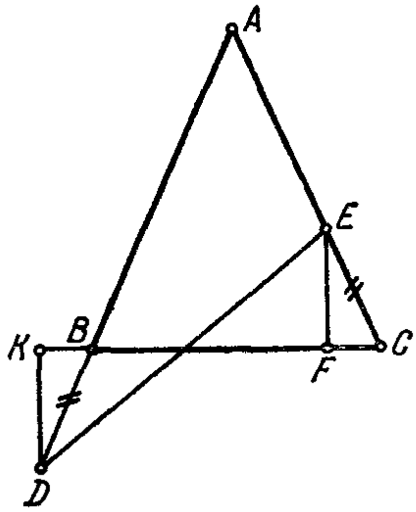
$$\frac{AM}{A_1M}\frac{BM}{B_1M}\frac{CM}{C_1M}=\frac{(S_2+S_3)(S_1+S_3)(S_2+S_1)}{S_1S_2S_3}\geq\frac{2\sqrt{S_2S_3}2\sqrt{S_1S_3}2\sqrt{S_2S_1}}{S_1S_2S_3}=8$$

Tenglik belgisi faqat M nuqta$-¿$ uchburchak medianalar kesishish nuqtasi bo'lganda bajariladi.

4-masala. Uchburchaklardan berilgan burchak , bu burchakka yopishgan tomonlar yig'indisi o'zgarmas bo'lganda , eng kichik perimetrga ega bo'lgan uchburchak turini toping.

Yechish. Ikkita $ABC(AB=AC)$ yonli va ADE uchburchaklarda (2-chizma) A burchak umumiy va

$$AB+AC=AD+AE \quad (1)$$



(2-chizma)

(1) tenglikdan $BD=EC$ ekanligini topamiz. B va C nuqtalardan BC to'g'ri chiziqqa DK va EF perpendikulyarlar o'tkazamiz. Osongina ko'rish mumkinki, BDK va CEF o'zaro teng uchburchaklar. Uchburchaklar tengligidan $BK=BC$, chunki $KF=BC$. Lekin KF DE ning BC to'g'ri chiziqdagi proeksiyasi, $KF<DE$ bo'lgani uchun

$$BC<DE \quad (2)$$

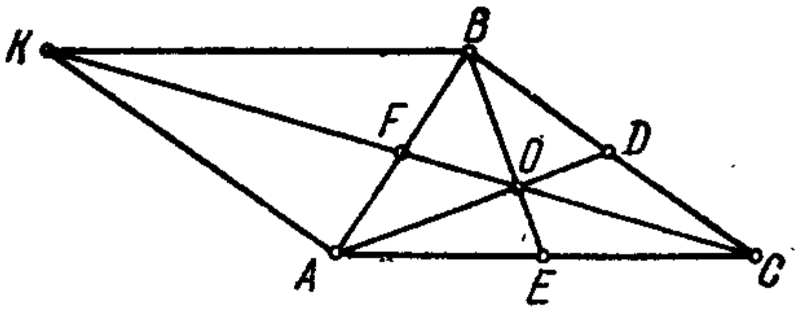
(1) va (2) tengliklarni hisobga olib $AB+AC+BC<AD+AE+DE$

ya'ni uchburchak teng yonli ekan.

5-masala. ABC uchburchakda BC va AC tomonlar mos ravishda a va b ga teng. Bu tomonlarga tushgan medianalar o'zaro perpendikulyar bo'lsa, quyidagi tengsizlik bajarilishini ko'rsating:

$$\frac{1}{2}<\frac{a}{b}<2$$

Isbot. ABC uchburchakda AD, BE, CF medianalar O nuqtada kesishsin (3-chizma). To'g'ri burchakli AOB uchburchakda $OF=0{,}5c$, bu yerda c$-AB$ tomon uzunligi. Shunday qilib, $CF=1{,}5c$ ga teng.



(3-chizma)

ABCuchburchakni $AKBC$parallelogrammgacha to'ldiramiz. Bu parallelogrammda diagonal $KC=3c$. Parallelogrammda diagonallarining kvadratlarining yig'indisi tomonlarining kvadratlarining yig'indisiga teng: $(3c)^2+c^2=2a^2+2b^2$ yoki $5c^2=a^2+b^2$ (1)

$|a-b|<c<a+b$ deb hisoblasak, quyidagicha shart hosil bo'ladi:

$$\begin{cases} 5(a+b)^2>a^2+b^2 \\ 5(a-b)^2<a^2+b^2 \end{cases} \qquad (2)$$

Birinchi tengsizlik va (2) dan ixtiyoriy $a$va $b$uchun bajariladigan :

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Bu yerdan $\frac{1}{2}<\frac{a}{b}<2$ kelib chiqadi.

6-masala. Quyidagi tengsizlikning bajarilishini isbotlang: $2\sqrt{(p-b)(p-c)}\leq a$. Bu yerda $a,b,c-¿$uchburchak tomonlari, $p-¿$uning yarim perimetri.

Isbot. Biz quyidagiga egamiz:

$2\sqrt{(p-b)(p-c)}=\sqrt{(a-b+c)(a+b-c)}=\sqrt{a^2-(b-c)^2}\leq a$. Ko'rinib turibdiki, tenglik belgisi faqat $b=c$ bo'lganda bajariladi.

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