THE APPROACH OF CONCURRENT PURSUIT IN A SECOND-ORDER DIFFERENTIAL GAME STRATEGY

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Abstract. *In this work is considered a differtial game of the second order, when control functions of the players satisfies geometric constraints. The proposed method substantiates the parallel approach strategy in this differential game of the second order. The new sufficient solvability conditions are obtained for problem of the pursuit.*

Keywords. *Differential game, geometric constraint, evader, pursuer, strategy of the parallel pursuit, acceleration.*

ПОДХОД ПАРАЛЛЕЛЬНОГО ПРЕСЛЕДОВАНИЯ В СТРАТЕГИИ ДИФФЕРЕНЦИАЛЬНОЙ ИГРЫ ВТОРОГО ПОРЯДКА

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Аннотация. *В работе рассматривается дифференциальная игра второго порядка при геометрических ограничениях на управления игроков. При этом предлагается стратегия параллельного преследования для преследователя и при помощи этой стратегии решается задача преследования.*

Ключевые слова: *дифференциальная игра, геометрическое ограничение, стратегия параллельного преследования, преследователь, убегающий, ускорения.*

Let $\mathbf{P}$ and $\mathbf{E}$ objects with opposite aim be given in the space R^n and their movements based on the following differential equations and initial conditions

$$\mathbf{P}: \quad \ddot{x} = u, \quad x_1 - kx_0 = 0, \quad |u| \le \alpha, \tag{1}$$

$$\mathbf{E}: \ddot{y} = v, \; y_1 - ky_0 = 0, \; |v| \le \beta, \qquad (2)$$

where $x, y, u, v \in \boldsymbol{R}^n$; x – a position of **P** object in the space $\boldsymbol{R}^n$, $x_0 = x(0),\ x_1 = \dot{x}(0)$ – its initial position and velocity respectively at $t = 0$; u – being a controlled acceleration of the pursuer, mapping $u:[0,\infty) \to \boldsymbol{R}^n$ and it is chosen as a measurable function with respect to time; we denote a set of all measurable functions $u(\cdot)$ such that satisfies the condition $|u| \le \alpha$ by $\boldsymbol{U}$. y – a position of **E** object in the space $\boldsymbol{R}^n$, $y_0 = y(0),\ y_1 = \dot{y}(0)$ – its initial position and velocity respectively at $t = 0$; v – being a controlled acceleration of the evader, mapping $v:[0,\infty) \to \boldsymbol{R}^n$ and it is chosen as a measurable function with respect to time; we denote a set of all measurable functions $v(\cdot)$ such that satisfies the condition $|v| \le \beta$ by $\boldsymbol{V}$.

Definition 1. For a trio of $(x_0, x_1, u(\cdot)), u(\cdot) \in \boldsymbol{U}$, the solution of the equation (1), that is, $x(t) = x_0 + x_1 t + \int_0^t \int_0^s u(\tau)\, d\tau ds$ is called a trajectory of the pursuer on interval $t \ge 0$.

Definition 2. For a trio of $(y_0, y_1, v(\cdot)),\ v(\cdot) \in \boldsymbol{V}$, the solution of the equation (2), that is, $y(t) = y_0 + y_1 t + \int_0^t \int_0^s v(\tau)\, d\tau ds$ is called a trajectory of the evader on interval $t \ge 0$.

Definition 3. The pursuit problem for the differential game (1) - (2) is called to be solved if there exists such control function $u^*(\cdot) \in \boldsymbol{U}$ of the pursuer for any control function $v(\cdot) \in V$ of the evader and the following equality is carried out at some finite time t^*

$$x(t^*) = y(t^*). \tag{3}$$

Definition 4. For the problem (1)-(2), time T is called a guaranteed pursuit time if it is equal to an upper boundary of all the finite values of pursuit time t^* satisfying the equality (3).

Definition 5. For the differential game (1) - (2), the following function is called Π-strategy of the pursuer ([3]-[4]):

$$u(v) = v - \lambda(v)\xi_0, \tag{4}$$

where $\xi_0 = \frac{z_0}{|z_0|}$, $\lambda(v,\xi_0) = (v,\xi_0) + \sqrt{(v,\xi_0)^2 + \alpha^2 - |v|^2}$,

(v,ξ_0) is a scalar multiplication of vectors v and ξ_0 in the space $\boldsymbol{R}^n$.

Property 1. If $\alpha \geq \beta$, then a function $\lambda(v,\xi_0)$ is continuous, nonnegative and defined for all v such that satisfies the inequality $|v| \leq \beta$.

Property 2. If $\alpha \geq \beta$, then the following inequality is true for the function $\lambda(v,\xi_0)$:

$$\alpha - |v| \leq \lambda(v,\xi_0) \leq \alpha + |v|.$$

Theorem. If one of the following conditions holds for the second order differential game (1) – (2), that is, 1. $\alpha = \beta$ and $k < 0$; or 2. $\alpha > \beta$ and $k \in R$, then by virtue of strategy (4) a guaranteed pursuit time becomes as follows

$$T=\begin{cases} (|z_0|k+\sqrt{|z_0|^2k^2+2|z_0|(\alpha-\beta)})/(\alpha-\beta), & agar\ k\neq 0\, va\, \alpha>\beta, \\ -1/k, & agar\ \ k<0\, va\, \alpha=\beta, \\ \sqrt{2|z_0|/(\alpha-\beta)}, & agar\ k=0\, va\, \alpha>\beta. \end{cases}$$

Proof. Suppose, let the pursuer choose the strategy in the form (4) when the evader chooses any control function $v(\cdot)\in V$. Then, according to the equations (1) – (2), we have the following Caratheodory's equation:

$$\ddot{z}=-\lambda(v(t))\xi_0, \quad \dot{z}(0)-kz(0)=0$$

Thus the following solution will be found by the given initial conditions

$$z(t)=z_0(kt+1)-\xi_0\int_0^t\int_0^s\lambda(v(\tau),\xi_0)d\tau ds$$

or

$$|z(t)|=|z_0|(kt+1)-\int_0^t\int_0^s\left((v(\tau),\xi_0)+\sqrt{(v(\tau),\xi_0)^2+\alpha^2-|v(\tau)|^2}\right)d\tau ds.$$

According to the properties 1- 2 , we will form the following inequalities

$$|z(t)|\le|z_0|(kt+1)-\int_0^t\int_0^s(\alpha-|v(\tau)|)d\tau ds \quad \Rightarrow$$

$$|z(t)|\le|z_0|(kt+1)+t^2(\beta-\alpha)/2.$$

If we say $f(t,a,k,\alpha,\beta)=a(kt+1)-\dfrac{t^2}{2}(\alpha-\beta),\ a=|z_0|$ (5), then we will check its properties

1. Let be $\alpha=\beta$.

1.1.If $k>0$, then the function $f(t,a,k,\alpha,\beta)=a(kt+1)$ is alwayscontinuous and isn't equal to zero.

1.2. If $k=0$, then the function $f(t,a,k,\alpha,\beta)=|z_0|$ is a linear function (Fig-2).

1.3. If $k < 0$, then the function (6) is decreasing and it equals to zero at $t^* = -\frac{1}{k}$

2. Let be $\alpha > \beta$. 2.1. If $k > 0$, then the function (6) is equal to zero at $T = \left(|z_0|k + \sqrt{|z_0|^2 k^2 + 2|z_0|(\alpha - \beta)}\right) / (\alpha - \beta)$.

In this case, a maximal time of unapproach is equal to $t_0 = |z_0|k / (\alpha - \beta)$ and therefore, a maximal distance between them equals to $f(t_0) = \left(2|z_0|(\alpha - \beta) + |z_0|^2 k^2\right) / 2(\alpha - \beta)$.

2.2. If $k < 0$, then the function (6) decreases monotonically, and this function turns to zero at time T as in the case .

2.3. If $k = 0$, then the function (6) becomes in the form

$f(t, a, k, \alpha, \beta) = a - \frac{t^2}{2}(\alpha - \beta)$ and the pursuit time equals to the followin:

$$T = \sqrt{\frac{2|z_0|}{\alpha - \beta}}$$

In conclusion, the relation (3) is true at some time t^* according to the inequality $|z(t)| \le |z_0|(kt + 1) + t^2(\beta - \alpha)/2$ and properties of (5), and it is determined that a relation $t^* \le T$ is correct, i.e., the pursuit problem is solved. Proved.

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