BA'ZI BIR XOSMAS INTEGRALLARNI EYLER INTEGRALLARI YORDAMIDA HISOBLASH

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Annotatsiya. Ushbu maqolada Betta va Gamma funksiyalarni xossalaridan foydalanib xosmas integrallarni yechish haqida ma'lumotlar keltirilgan. Murakkab xosmas integrallarni Eyler integrallari yordamida yeshish usullari qarab chiqilgan.

Kalit so'zlar: betta funksiya, gamma funksiya, xosmas integral,integral.

CALCULATION OF SOME IMPROPER INTEGRALS USING EULER INTEGRALS

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Abstract. This article provides information on solving nonlinear integrals using the properties of Betta and Gamma functions. Methods for solving complex nonlinear integrals using Euler integrals are considered.

Key words: betta function, gamma function, hosmas integral, integral

Ushbu

$$\int_0^1 x^{a-1}(1-x)^{b-1}dx \qquad (a>0, b>0) \tag{1}$$

xosmas integral *Betta funksiya* yoki *1-tur Eyler integrali* deyiladi va $B(a,b)$ kabi belgilanadi, ya'ni

$$B(a,b)=\int_0^1 x^{a-1}(1-x)^{b-1}dx \quad . \qquad (2)$$

Integral ostidagi funksiya uchun:

1) $a<0, b\geq 0$ bo'lganda $x=0$ nuqta;

2) $a\geq 1, b<1$ bo'lganda $x=1$ nuqta;

3) $a<1, b<1$ bo'lganda $x=0$ va $x=1$ nuqtalar maxsus nuqtalar bo'ladi.

Demak, (1) integral parametrga bog'liq bo'lgan xosmas integraldir.

Betta funksiya quyidagi xossalarga ega.

1-xossa. (1) integral

$M=\{(a;b)\in R^2 : a\in(0;+\infty), b\in(0;+\infty)\}$

to'plamda yaqinlashuvchi.

2-xossa. (1) integral $M_0=\{(a;b)\in R^2 : a\in[a_0;+\infty), b\in[b_0;+\infty)\}, a_0>0, b_0>0$ to'plamda tekis yaqinlashuvchi, lekin M to'plamda esa notekis yaqinlashuvchi.

3-xossa. $B(a,b)$ funksiya M to'plamda uzluksiz funksiyadir.

4-xossa. $\forall(a,b)\in M$ uchun $B(a,b)=B(b,a)$ (simmetrik) bo'ladi.

5-xossa. $B(a,b)$ funksiya quyidagicha ham ifodalanadi:

$$B(a,b)=\int_0^{+\infty}\frac{t^{a-1}}{(1+t)^{a+b}}dt$$

Isbot:

$$B(a;b)=\int_0^1 x^{a-1}(1-x)^{b-1}dx = \left| \frac{x}{1-x}=t \quad x=t-tx \quad x+tx=t \quad x(1+t)=t \quad x=\frac{t}{1+t} \quad dx=\frac{(1+t)-t}{(1+t)^2}dt=\frac{dt}{(1+t)^2} \right| =$$

$$\int_0^{\infty}\left(\frac{t}{1+t}\right)^{a-1}\cdot\left(\frac{1}{1+t}\right)^{b-1}\cdot\frac{dt}{(1+t)^2}=\int_0^{\infty}\frac{t^{a-1}}{(1+t)^{a-1}}\cdot\frac{1}{(t+t)^{b-1}}\cdot\frac{dt}{(1+t)^2}=$$

$$¿\int_0^{\infty}\frac{t^{a-1}}{(1+t)^{a+b-2+2}}dt=\int_0^{\infty}\frac{t^{a-1}}{(1+t)^{a+b}}dt.$$

xossa isbotlandi.

6-xossa. $\forall (a,b)\in M_1 = ¿¿$ uchun

$$B(a,b)=\frac{b-1}{a+b-1}\cdot B(a,b-1)$$

$B(a,1-a)=\dfrac{\pi}{\sin a\pi}\ (0<a<1)$, xususiy holda $a=\dfrac{1}{2}$ bo'lganda,

7-xossa.

$$B=\left(\frac{1}{2},\frac{1}{2}\right)=\pi$$

Gamma funksiya va uning xossalari.

Ushbu

$$\int_0^{+\infty} x^{a-1}e^{-x}dx \qquad (a>0) \tag{3}$$

xosmas integral *Gamma funksiya* yoki 2-tur *Eyler integrali* deyiladi va quyidagicha belgilanadi, yani

$$\Gamma(a)=\int_{0}^{+\infty} x^{a-1}e^{-x}dx \qquad (a>0) \tag{4}$$

Integral ostidagi funksiya uchun:

1) $a<1$ bo'lganda $x=0$ nuqta maxsus nuqta;
2) $a>1$ bo'lganda (3) integral yaqinlashuchi;
3) $a\leq 0$ bo'lganda (3) integral uzoqlashuvchi;

Gamma funksiya quydagi xossalarga ega.

1-xossa.

$$\Gamma(a)=\lim_{n\to\infty} n^{a}\frac{1\cdot 2\cdot 3\cdot \text{¿}\cdot (n-1)}{a(a=1)\cdot \text{¿}\cdot (a+n+1)}. \tag{5}$$

(5) formula *Eyler-Gauss* formulasi deyiladi.

2-xossa. (3) integral $\forall\, a\in[a_0,b_0]$ $(0<a_0<b_0<+\infty)$ oraliqda tekis yaqinlashuvchi, $a\in(0,+\infty)$ da esa notekis yaqinlashuvchi.

3-xossa. $\Gamma(a)$ funsiya $(0;+\infty)$ oraliqda uzluksiz va barcha tartibdagi uzluksiz hosilalarga ega, ya'ni

$$\Gamma^{(n)}(a)=\int_{0}^{+\infty} x^{a-1}e^{x}(\ln x)^{n}dx \ \ (n\in N).$$

4-xossa. $\Gamma(a+1)=a\Gamma(a) \ \ (a>0)$.

5xossa. $\Gamma(n+1)=n\,!$

6-xossa. $B(a,b)\equiv\text{¿}\frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}.\text{¿}$

7xossa. $\Gamma(a)\Gamma(1-a)=B(a,1-a)=\frac{\pi}{\sin a\pi}, \quad 0<a<1.$

8-xossa. $\Gamma(n+\frac{1}{2})=\frac{(2n-1)!!}{2^{n}}\sqrt{\pi}, \ \ n\in N.$

9-xossa. Lejandr formulasi: $\Gamma(a)\Gamma(a+\frac{1}{2})=\frac{\sqrt{\pi}}{2^{2a-1}}\Gamma(2a).$

Endi yuqoridagi betta va gamma funksiyalar yordamida ba’zi bir xosmas integrallarni hisoblashga doir bir nechta misollarni ko’rib chiqamiz.

1-misol. Eyler integralidan foydalanib quydagi xosmas integralni hisoblang.

$$\int_0^2 \frac{dx}{\sqrt[3]{x^2(2-x)}}$$

Yechilishi. Bu integralda $\frac{x}{2-x}=t$ almashtirish orqali quyidagi integralni hosil qilin, keyin uning qiymatini hisoblaymiz.

$$\left[\frac{x}{2-x}=t,\right]\left[x=2t-tx,\right]\left[x\cdot(1+t)=2t\right]\left[x=\frac{2t}{1+t}\right] \Rightarrow$$

$$\int_0^\infty \frac{2\,dt}{(1+t)^2\cdot\sqrt[3]{(\frac{2t}{1+t})^2\cdot(2-\frac{2t}{1+t})}}=\int_0^\infty \frac{2\,dt}{(1+t)^2\cdot\sqrt[3]{\frac{4t^2}{(1+t)^2}\cdot\frac{2}{1+t}}}=$$

$$=\int_0^\infty \frac{2\,dt}{(1+t)^2\cdot\frac{2}{t+1}\cdot t^{\frac{2}{3}}}=[\textit{qisqartirishlarni}\ \ \textit{ama}\lg a\ \ \textit{oshirib}]=\int_0^\infty \frac{t^{\frac{-2}{3}}}{1+t}dt=\int_0^\infty \frac{t^{1-\frac{2}{3}-1}}{1+t}dt=$$

$$=\int_0^\infty \frac{t^{\frac{1}{3}-1}}{(1+t)^{\frac{1}{3}+\frac{2}{3}}}dt=B(\frac{1}{3};\frac{2}{3})=[\textit{Betta funksiyaning } 7-\textit{xossasiga asosan}]=\frac{\pi}{\sin\frac{\pi}{3}}=\frac{\pi}{\frac{\sqrt{3}}{2}}=\frac{2\pi}{\sqrt{3}}$$

Foydalanilgan adabiyotlar ro’yxati

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