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NUMERICAL SOLUTION OF THE HEAT DISTRIBUTION EQUATION USING THE GRID METHOD.

Abstract. In this article presents an algorithm for the numerical solution heat equations using the mesh method on a rectangular area and a program in the Delphi-7 language is created based on this algorithm.

Key words: Algorithm, grid method, scheme, program, Delphi-7, sistem.

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ЧИСЛЕННОЕ РЕШЕНИЕ УРАВНЕНИЯ РАСПРЕДЕЛЕНИЯ ТЕПЛА С ИСПОЛЬЗОВАНИЕМ МЕТОДА СЕТОК.

Аннотация. В этой статье приведен алгоритм численного решения уравнения теплопроводности методом сетка на прямоугольной области и на основе этого алгоритма создана программа числена решающая уравнения теплопроводности на прямоугольной области на языке Delphi-7.

Ключевые слова: Алгоритм, метод сетка, схема, программа, Delphi-7, система.

At the beginning of the 21st century, with the rapid development of science and technology, information and communication technologies, and globalization, there is a need to improve the content of mathematics and the methodological system, forms, methods and tools of teaching, to develop the pedagogical potential of students of higher educational institutions in our country's continuous education system based on the integration of programming programs with information technologies.

Nowadays, a mathematics teacher must guide his students towards modern thinking, reading, learning, and activities that create their own voluntary activities in the current conditions, not based on previously assigned activities.

Therefore, in our opinion, a modern teacher should not only study his/her own subject, but also learn to effectively use programming programs based on pedagogical and information technologies in the educational process. At the same time, our main task is to prepare future mathematics teachers who are able to innovate (innovate) the educational process, have the ability to be researchers, creative seekers, and quickly adapt to the demands of the time. [4].

Nowadays, a sufficiently large number of hours are allocated for independent study in the curriculum for students to fully master the subjects. Students need to work more on themselves in order to further consolidate the theoretical and practical knowledge they have acquired in the classroom.

We will give an example of students working independently.

1. The problem is posed $Q=(0,T)\times\Omega$ in the field $\frac{\partial u}{\partial t}=A^2(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2})+f(x,y,t),\ \ z=(x,y)\in\Omega$ (1) heat equation $u(z,0)=\varphi(z),\ z\in\overline{\Omega},$ (2) $u(z,t)=\psi(z,t),\ z\in\partial\Omega, 0\le t\le T$ (3) satisfying the initial and boundary conditions $u(x,y,t)$ find a solution. Here $\Omega=\{z=(x,y): a<x<b,\ c<y<d\}$, $\partial\Omega$-Ω the boundary of the field. $A=const>0$, $\overline{\Omega}=\Omega\cup\partial\Omega$, $f(z,t),\varphi(z),\psi(z,t)$- given continuous functions. (1)-(3) that the issue has been raised correctly. [2].

2. Solution algorithm. Data to be entered into the program:

1. Immutable: A,T,a,b,c,d;

2. Functions: $f(x,y,t),\varphi(x,y),\psi(x,y,t)$;

3. t,x,y number of divisions according to N_t,N_x,N_y.

$f(x,y,t),\varphi(x,y),\psi(x,y,t)$ functions are elementary functions, real numbers, arithmetic operations in the program: + add, - subtraction, / - divided by, * - multiply is entered using. p≈3.14 The number is entered with the symbol pi. The table below shows the elementary functions and how to include them in the program (see Table 1).

Table 1.

Table of elementary functions to be included in the program

No.	Elementary functions	Introducing elementary functions into the program	No.	Elementary functions	Introducing elementary functions into the program
1	y=|x|	mod (x)	17	y= ln x	ln (x)
2	y=[x]	butun (x)	18	y= lg x	lg (x)
3	y={x}	kasr (x)	19	y= log x	log (x)
4	$y=x^n$, n€N	dar (x:n)	20	y= sh x	sh (x)
5	$y=\sqrt[n]{x}$, n€N	ildiz (x:n)	21	y= th x	th (x)
6	y= sinx	sin (x)	22	y= sch x	sch (x)
7	y= cos x	cos (x)	23	y= ch x	ch (x)
8	y=tg x	tg (x)	24	y= cth x	cth (x)
9	y=ctg x	ctg (x)	25	y= csch x	csch (x)
10	y= sec x	sec (x)	26	y= arsh x	arsh (x)
11	y=cosec x	cosec (x)	27	y= arch x	arch (x)
12	y= arcsin x	arcsin (x)	28	y= arth x	arth (x)
13	y= arccos x	arccos (x)	29	y= arcth x	arcth (x)
14	y= arctg x	arctg (x)	30	y= arcsec x	arcsec (x)
15	y= arcctg x	arcctg (x)	31	y= arccsc x	arccsc (x)
16	$y= a^x$	kurs (a:x)	32	$y= e^x$	e (x)

For example $f(x,y,t)=\sqrt[3]{x^2+y^2}+e^{5t^3}$ The function is introduced into the program as follows: root (dar(X:2)+dar(Y:2):3)+e (5*dar(T:3)).

(1)-(3) We will solve the problem using the grid method. $[0;T]$ cut $t_n = n\tau;\ n=\overline{0,N_t};\ \tau=\dfrac{T}{N_t}$ with dots N_t We divide it into equal parts. Similarly, $[a;b]$ cut $x_i = a+ih_x;\ i=\overline{0,N_x};\ h_x=\dfrac{b-a}{N_x}$ with dots N_x We divide it into equal parts. $[c;d]$ cut $y_j = c+jh_y;\ j=\overline{0,N_y};\ h_y=\dfrac{d-c}{N_y}$ with dots N_y We divide it into equal parts. $z_{ij}^n=(x_i,y_j,t_n)$ - it is called a net knot. z_{ij}^n in a knot $u(x,y,t)$ approximate value of the solution u_{ij}^n We denote it by the symbol, that is, $u(x_i,y_j,t_n)\approx u_{ij}^n$. We approximate the particular derivatives as follows::

$$
\begin{aligned}
&\frac{\partial u}{\partial t}(x_i,y_j,t_{n+1})=\frac{u_{ij}^{n+1}-u_{ij}^{n}}{\tau}+O(\tau);\\
&\frac{\partial^2 u}{\partial x^2}(x_i,y_j,t_{n+1})=\frac{u_{i-1j}^{n+1}-2u_{ij}^{n+1}+u_{i+1j}^{n+1}}{h_x^2}+O(h_x^2);\\
&\frac{\partial^2 u}{\partial y^2}(x_i,y_j,t_{n+1})=\frac{u_{ij-1}^{n+1}-2u_{ij}^{n+1}+u_{ij+1}^{n+1}}{h_y^2}+O(h_y^2).
\end{aligned}
\tag{4}
$$

(4) Using the equalities, we form the following implicit scheme for equation (1):

$$
\begin{aligned}
&\frac{u_{ij}^{n+1}-u_{ij}^{n}}{\tau}=A^2\left(\frac{u_{i-1j}^{n+1}-2u_{ij}^{n+1}+u_{i+1j}^{n+1}}{h_x^2}+\frac{u_{ij-1}^{n+1}-2u_{ij}^{n+1}+u_{ij+1}^{n+1}}{h_y^2}\right)+f(x_i,y_j,t_{n+1}),\\
&n=\overline{0,N_t-1},\ i=\overline{1,N_x-1},\ j=\overline{1,N_y-1}.
\end{aligned}
\tag{5}
$$

(5) approximation order of the scheme $O(\tau+h_x^2+h_y^2)$. (5) We write the differential equations in the following form::

$$
\begin{aligned}
&-\frac{A^2\tau}{h_x^2}u_{i-1j}^{n+1}-\frac{A^2\tau}{h_y^2}u_{ij-1}^{n+1}+\left[1+2A^2\tau\left(\frac{1}{h_x^2}+\frac{1}{h_y^2}\right)\right]u_{ij}^{n+1}-\\
&-\frac{A^2\tau}{h_x^2}u_{i+1j}^{n+1}-\frac{A^2\tau}{h_y^2}u_{ij+1}^{n+1}=\tau f(x_i,y_j,t_{n+1})+u_{ij}^{n},\\
&n=\overline{0,N_t-1},\ i=\overline{1,N_x-1},\ j=\overline{1,N_y-1}.
\end{aligned}
\tag{6}
$$

We approximate the initial and boundary conditions as follows:

$$\begin{aligned}
&u_{ij}^{0}=\varphi(x_i,y_j),\quad z_{ij}=(x_i,y_j)\in\overline{\Omega_h},\\
&u_{ij}^{n+1}=\varphi(x_i,y_j,t_{n+1}),\quad z_{ij}=(x_i,y_j)\in\partial\Omega_h,\\
&0<t_{n+1}<\tau N_t,\quad n=\overline{0,N_t-1}.
\end{aligned}\tag{7}$$

Here $\Omega_h=\left\{z_{ij}=(x_i,y_j): a<x_i<b,\, c<y_j<d\right\}$.

(6)-(7) the problem is stable (see [2], p. 181, Theorem 2). relatively formed (6) The system of linear equations (7) is a closed system together with the system of linear equations (6). The condition of diagonal dominance is fulfilled in the system of linear equations (6)[5]. Therefore, we solve the system of linear equations (6) by the Seidel method[5]. Based on this algorithm, a program was created in the Delphi-7 programming environment to numerically solve problems (1)-(3) in a rectangular area and plot the solution graph. The performance of the created program was verified in computational experiments.

3. Numerical calculations. Problem. $Q=\{0<x<5,\ 0<y<5,\ 0<t<5\}$ in the field

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}-3\tag{8}$$

The heat equation

$$u(x,y,0)=x^2+y^2\tag{9}$$

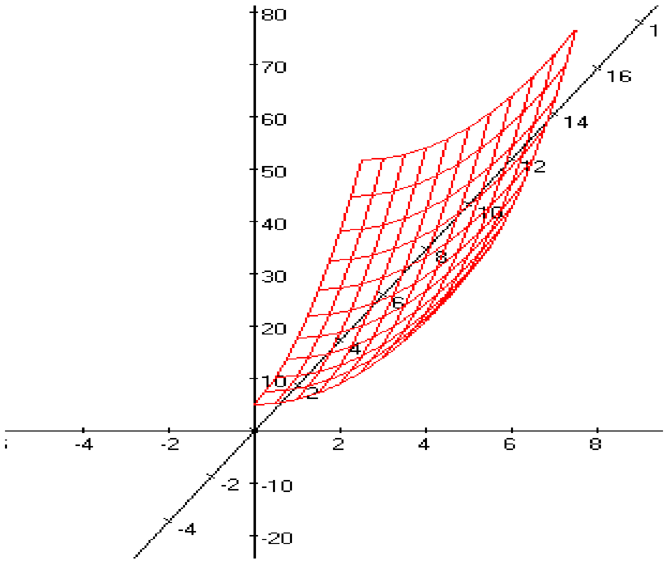
the initial condition and

$$\begin{aligned}
&u(0,y,t)=y^2+t,\ 0\le y\le 5,\ 0\le t\le 5;\\
&u(5,y,t)=y^2+t+25,\ 0\le y\le 5,\ 0\le t\le 5;\\
&u(x,0,t)=x^2+t,\ 0\le x\le 5,\ 0\le t\le 5;\\
&u(x,5,t)=x^2+t+25,\ 0\le x\le 5,\ 0\le t\le 5.
\end{aligned}\tag{10}$$

satisfying boundary conditions $u(x,y,t)$ find a solution. (8)-(10) clear solution to the problem $u(x,y,t)=x^2+y^2+t$.

Solution. Below $T=5$ да, $N_x=10, N_y=10, N_t=20$ when (8)-(10) an approximate solution to the problem is presented at the nodes.

u[0,0]=5,00	u[1,0]=5,25	u[2,0]=6,00	u[3,0]=7,25	u[4,0]=9,00
u[5,0]=11,25	u[6,0]=14,00	u[7,0]=17,25	u[8,0]=21,00	u[9,0]=25,25
u[10,0]=30,00	u[0,1]=5,25	u[1,1]=5,50	u[2,1]=6,25	u[3,1]=7,50
u[4,1]=9,25	u[5,1]=11,50	u[6,1]=14,25	u[7,1]=17,50	u[8,1]=21,25
u[9,1]=25,50	u[10,1]=30,25	u[0,2]=6,00	u[1,2]=6,25	u[2,2]=7,00
u[3,2]=8,25	u[4,2]=10,00	u[5,2]=12,25	u[6,2]=15,00	u[7,2]=18,25
u[8,2]=22,00	u[9,2]=26,25	u[10,2]=31,00	u[0,3]=7,25	u[1,3]=7,50
u[2,3]=8,25	u[3,3]=9,50	u[4,3]=11,25	u[5,3]=13,50	u[6,3]=16,25
u[7,3]=19,50	u[8,3]=23,25	u[9,3]=27,50	u[10,3]=32,25	u[0,4]=9,00
u[1,4]=9,25	u[2,4]=10,00	u[3,4]=11,25	u[4,4]=13,00	u[5,4]=15,25
u[6,4]=18,00	u[7,4]=21,25	u[8,4]=25,00	u[9,4]=29,25	u[10,4]=34,00
u[0,5]=11,25	u[1,5]=11,50	u[2,5]=12,25	u[3,5]=13,50	u[4,5]=15,25
u[5,5]=17,50	u[6,5]=20,25	u[7,5]=23,50	u[8,5]=27,25	u[9,5]=31,50
u[10,5]=36,25	u[0,6]=14,00	u[1,6]=14,25	u[2,6]=15,00	u[3,6]=16,25
u[4,6]=18,00	u[5,6]=20,25	u[6,6]=23,00	u[7,6]=26,25	u[8,6]=30,00
u[9,6]=34,25	u[10,6]=39,00	u[0,7]=17,25	u[1,7]=17,50	u[2,7]=18,25
u[3,7]=19,50	u[4,7]=21,25	u[5,7]=23,50	u[6,7]=26,25	u[7,7]=29,50
u[8,7]=33,25	u[9,7]=37,50	u[10,7]=42,25	u[0,8]=21,00	u[1,8]=21,25
u[2,8]=22,00	u[3,8]=23,25	u[4,8]=25,00	u[5,8]=27,25	u[6,8]=30,00
u[7,8]=33,25	u[8,8]=37,00	u[9,8]=41,25	u[10,8]=46,00	u[0,9]=25,25
u[1,9]=25,50	u[2,9]=26,25	u[3,9]=27,50	u[4,9]=29,25	u[5,9]=31,50
u[6,9]=34,25	u[7,9]=37,50	u[8,9]=41,25	u[9,9]=45,50	u[10,9]=50,25
u[0,10]=30,00	u[1,10]=30,25	u[2,10]=31,00	u[3,10]=32,25	u[4,10]=34,00
u[5,10]=36,25	u[6,10]=39,00	u[7,10]=42,25	u[8,10]=46,00	u[9,10]=50,25
u[10,10]=55,00				



$u(x,y,t)$ solution graph.

In conclusion, for mathematics students in higher education institutions, the teacher's creative approach to demonstrating the interdisciplinary integration of "Differential Equations" and "Programming" will increase students' interest in the subject.

Examples for independent solution.

1) $Q=\{0<x<5,\ 0<y<5, 0<t<2\ \}$ in the field

$$\frac{\partial u}{\partial t}=2\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-8+xy$$

heat equation

$$u(x,y,0)=x^2+y^2$$

the initial condition and

$$u(0,y,t)=y^2,\ 0\le y\le 5, 0\le t\le 2;$$
$$u(5,y,t)=5yt+y^2+25,\ 0\le y\le 5, 0\le t\le 2;$$
$$u(x,0,t)=x^2,\ 0\le x\le 5, 0\le t\le 2;$$
$$u(x,5,t)=x^2+5xt+25,\ 0\le x\le 5, 0\le t\le 2.$$

satisfying boundary conditions $u(x,y,t)$ find a solution.

2) $Q=\{0<x<2,\ 0<y<5, 0<t<2\ \}$ in the field

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+x^2y-2yt$$

heat equation

$$u(x, y, 0) = 0$$

the initial condition and

$$\begin{aligned} &u(0, y, t) = 0,\ 0 \le y \le 5,\ 0 \le t \le 2; \\ &u(2, y, t) = 4yt,\ 0 \le y \le 5,\ 0 \le t \le 2; \\ &u(x, 0, t) = 0,\ 0 \le x \le 2,\ 0 \le t \le 2; \\ &u(x, 5, t) = 5x^2 t,\ 0 \le x \le 2,\ 0 \le t \le 2. \end{aligned}$$

satisfying boundary conditions $u(x, y, t)$ find a solution.

3) $Q = \{0 < x < \pi/2,\ 0 < y < \pi,\ 0 < t < 5\}$ in the field

$$\frac{\partial u}{\partial t} = 3\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + (1 - 6t)\sin x \cos y$$

heat equation

$$u(x, y, 0) = 0$$

the initial condition and

$$\begin{aligned} &u(0, y, t) = 0,\ 0 \le y \le \pi,\ 0 \le t \le 5; \\ &u(\pi/2, y, t) = t\cos y,\ 0 \le y \le \pi,\ 0 \le t \le 5; \\ &u(x, 0, t) = t\sin x,\ 0 \le x \le \pi/2,\ 0 \le t \le 5; \\ &u(x, \pi, t) = -t\sin x,\ 0 \le x \le \pi/2,\ 0 \le t \le 5. \end{aligned}$$

satisfying boundary conditions $u(x, y, t)$ find a solution.

4) $Q = \{0 < x < 5,\ 0 < y < 5,\ 0 < t < 3\}$ in the field

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{(x^2 + y^2)t}{\sqrt{(1 + t^2)^3}} + \frac{4}{\sqrt{1 + t^2}}$$

heat equation

$$u(x, y, 0) = -(x^2 + y^2)$$

the initial condition and

$$u(0,y,t) = -\frac{y^2}{\sqrt{1+t^2}},\ 0 \le y \le 5,\ 0 \le t \le 3;$$

$$u(5,y,t) = -\frac{y^2+25}{\sqrt{1+t^2}},\ 0 \le y \le 5,\ 0 \le t \le 3;$$

$$u(x,0,t) = -\frac{x^2}{\sqrt{1+t^2}},\ 0 \le x \le 5,\ 0 \le t \le 3;$$

$$u(x,5,t) = -\frac{x^2+25}{\sqrt{1+t^2}},\ 0 \le x \le 5,\ 0 \le t \le 3.$$

satisfying boundary conditions $u(x,y,t)$ find a solution.

5) $Q = \{0 < x < 2,\ 0 < y < 2,\ 0 < t < 4\}$ in the field

$$\frac{\partial u}{\partial t} = 4\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + \frac{x^2+y^2}{2\sqrt{t+1}} - 16\sqrt{t+1}$$

heat equation

$$u(x,y,0) = x^2 + y^2$$

the initial condition and

$$u(0,y,t) = y^2\sqrt{t+1},\ 0 \le y \le 2,\ 0 \le t \le 4;$$
$$u(2,y,t) = (4+y^2)\sqrt{t+1},\ 0 \le y \le 2,\ 0 \le t \le 4;$$
$$u(x,0,t) = x^2\sqrt{t+1},\ 0 \le x \le 2,\ 0 \le t \le 4;$$
$$u(x,2,t) = (4+x^2)\sqrt{t+1},\ 0 \le x \le 2,\ 0 \le t \le 4.$$

satisfying boundary conditions $u(x,y,t)$ find a solution.

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